Computational Statistics and Data Analysis ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

Semi-parametric estimation of Brown–Proschan preventive maintenance effects and intrinsic wear-out

L. Doyen

University Grenoble Alpes, LJK, F-38000 Grenoble, France

ARTICLE INFO

Article history:
Received 19 June 2013
Received in revised form 5 February 2014
Accepted 26 February 2014
Available online xxxx

Keywords:
Reliability
Imperfect maintenance
Aging
Maintenance efficiency

ABSTRACT

A system subject to corrective and preventive maintenance actions is considered. Corrective Maintenance (CM) is done at unpredictable random times and is assumed to have As Bad As Old (ABAO) effects. Preventive Maintenance (PM) is supposed to be done at deterministic predetermined times and to follow a Brown–Proschan (BP) model, i.e., each PM is As Good As New (AGAN) with probability p and ABAO with probability $1 - p$. In this context a semi-parametric estimation method is proposed: nonparametric estimation of the first time to failure distribution and parametric estimation of the maintenance effect p. This work is original in considering that BP effects (ABAO or AGAN) are unknown or unobserved.

1. Introduction

A major issue for industrial systems is the joint management of aging and maintenance. Efficient maintenance and controlled aging allow the extension of the equipment's operating life. There are several kinds of maintenance. Corrective Maintenance (CM; also called repair) is carried out after a failure and is intended to restore the system's functionality. Preventive Maintenance (PM) is carried out when the system is operating and is intended to slow down the wear process and reduce the rate of occurrence of failures.

The basic assumptions of maintenance efficiency are known as minimal repair or As Bad As Old (ABAO) and perfect repair or As Good As New (AGAN). In the ABAO case, each maintenance leaves the system in the state it was in before maintenance. In the AGAN case, each maintenance is perfect and leaves the system as if it were new. The corresponding random processes are the Non Homogeneous Poisson Process (NHPP) and the Renewal Process (RP) respectively. Obviously reality falls between these two extremes: standard maintenance improves the reliability of the system, without necessarily completely renewing it. This is known as imperfect maintenance.

Many papers, for example Huang and Yen (2009), Sheu and Chang (2009), Wu and Zuo (2010), Sheu et al. (2010) use imperfect maintenance models in order to optimize PM scheduling according to cost criteria. However, both cost function and maintenance times are functions of the model parameters. For practical systems, the model parameters are not known, so they must be estimated. Yet, few authors have worked on the statistical analysis of PM and CM processes. Among the most recent publications, Syamsundar and Naikan (2011), Babykina and Couallier (2012), Fuqing and Kumar (2012), Pulcini (2013) and Yu et al. (2013) proposed statistical results for imperfect repair models. Liu et al. (2012) developed interesting statistical results for imperfect PM–CM models. Gilardoni et al. (2013) studied the nonparametric estimation of the (cost) optimal PM-period for the AGAN PM–ABAO CM model. Doyen and Gaudoin (2011) developed a general framework for modeling and

E-mail address: laurent.doyen@imag.fr.

http://dx.doi.org/10.1016/j.csda.2014.02.022
0167-9473/

assessment of the CM efficiency and planned PM. Li and Hanson (2014) proposed a Bayesian semi-parametric estimation method (including co-variables) for Kijima (type I or type II) CM models.

Many imperfect maintenance models have been proposed (see, for example, a review in Pham and Wang (1996)). The present paper focuses on the model proposed by Brown and Proschan (1983), denoted as BP in the following. The BP model was first introduced for systems submitted only for repair actions and assumes that:

- With probability p, system state after a repair is AGAN.
- With probability $1 - p$, system state after a repair is ABAO.
- The repair effects (AGAN or ABAO) are mutually independent, and independent of already observed failure times.

The random effect of repair actions can be represented by the variables B:

$$B_i = \begin{cases} 1 & \text{if the } i\text{th repair is AGAN,} \\ 0 & \text{if the } i\text{th repair is ABAO.} \end{cases}$$

Previous assumptions imply that the random variables $\{B_i\}_{i\geq 1}$ are mutually independent and identically Bernoulli-distributed with parameter p.

Authors have often supposed that the effect of each repair (B) is known and have developed statistical methods for estimating the first time to failure distribution (which represents wear-out of the new system in the absence of failure). Most of these papers (for example Whitaker and Samaniego (1989), Hollander et al. (1992), Kvam et al. (2002), Sethuraman and Hollander (2009)) focus on nonparametric estimation methods. In practice, maintenance effects are generally unknown: parameter p only represents the degree of efficiency of maintenance actions. To our knowledge, only a few papers deal with the BP model with unknown repair effects. The following three papers only considered CM. Lim (1998) estimated the parameters of the first time to failure distribution and repair efficiency with the Expectation–Maximization (EM) algorithm. Doyen (2011) generalized this approach and considered maximum likelihood estimation; the behavior of the BP CM models when repair effects are unknown was also derived. Lim and Lie (2000) used a SEM algorithm to estimate the parameters of a generalized BP model that allows first-order dependency between two consecutive repair effects, and they assumed that only some repair effects were unknown. Langseth and Lindqvist (2003) generalized the BP model for imperfect preventive maintenance. They estimated the parameters of the model (including the maintenance efficiency parameter) with the likelihood function. Doyen (2012) proposed both direct maximum likelihood and EM methods for BP PM and ABAO CM models. All of the above work on the BP model with unknown maintenance effects considered parametric estimation.

We propose a semi-parametric estimation method for the BP PM–ABAO CM model: nonparametric estimation of the first time to failure distribution and parametric estimation of the maintenance effect. The originality of this work is in considering that BP effects (ABAO or AGAN) are unknown. Doyen (2011) highlights the specific behavior of the BP failure process due to considering unknown maintenance effects. In fact, each maintenance effect is neither AGAN nor ABAO but has a probability to be AGAN or ABAO. Next, these BP unobserved random maintenance effects can be assessed, which provide individual differentiate estimations of PM efficiency. These PM efficiency estimations do not correspond to perfect or minimal maintenance effects but to some intermediate imperfect maintenance effects. The more probable AGAN PM is, the more efficient the PM is, and reversely. The result can be used to estimate maintenance efficiencies for PM and CM times corresponding to completely unknown PM effects intermediate between AGAN and ABAO. The proposed estimation method can be applied to a single system or to multiple independent systems with the same first time to failure distribution and different or identical probability of perfect PM.

Notations and the model are defined in Section 2. Section 3 generalizes the results of Doyen (2012) for the BP PM–ABAO CM model to multiple independent systems: likelihood definition, individual PM efficiency estimators, EM algorithm. A semi-parametric estimation method is derived from the EM method in Section 3. The corresponding algorithm is detailed in Appendix B. Finally, results are applied to simulated data in Section 4.

2. Notations and assumptions

K independent systems are considered. They are new at time 0. In the absence of maintenance, they assume the same failure rate $\lambda(t)$, called initial intensity. It represents the systems' intrinsic wear-out. Let $\Lambda(t)$ denote the corresponding cumulative intensity: $\Lambda(t) = \int_0^t \lambda(s)\,ds$.

The kth system, for $k \in \{1, \ldots, K\}$, is assumed to be preventively maintained at predetermined deterministic times $\{\tau_m^k\}_{1\leq m\leq m^k}$. The associated counting process is denoted by $\{m_t^k\}_{t\geq 0}$. Every time of PM is known and the duration of PM is not taken into account. PM effects are assumed to follow the BP model. p_k is the probability that a PM renews the system k, it represents the average global PM efficiency of system k.

CM is carried out at unpredictable random times, aiming to quickly restore the system to working order (in contrast with PM actions that are planned in advance in order to check and improve system reliability). The effects of CM are assumed to be ABAO. The CM duration is also not taken into account.

Failure times T_i^k (or equivalently CM times) are supposed to be observed between $c^k \geq 0$ and $T^k > c^k$. c^k is supposed to be a deterministic time, this is a left deterministic censoring time. But T^k can be a deterministic (time truncated data) or random time. For example, it can be the nth failure time, where n is deterministic (failure truncated data), or the mth maintenance time, where m is deterministic. The counting process associated with CM times is denoted by $\{N_t^k\}_{t\geq c^k}$.

Please cite this article in press as: Doyen, L., Semi-parametric estimation of Brown–Proschan preventive maintenance effects and intrinsic wear-out. Computational Statistics and Data Analysis (2014), http://dx.doi.org/10.1016/j.csda.2014.02.022