Computational Statistics and Data Analysis 76 (2014) 76–94

Contents lists available at ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

Modeling tails of aggregate economic processes in a stochastic growth model

Stéphane Auray [a,b,*], Aurélien Eyquem [c], Frédéric Jouneau-Sion [c]

[a] CREST-Ensai, Université du Littoral Côte d'Opale (EQUIPPE), France

[b] CIRPEE, Canada

[c] Université de Lyon, Université Lumière Lyon 2, CNRS (GATE UMR 5824), France

ARTICLE INFO

Article history:
Received 15 February 2013
Received in revised form 10 February 2014
Accepted 11 February 2014
Available online 20 February 2014

Keywords:
Economic growth
Unified growth theory
Heteroskedasticity
Fat tails

ABSTRACT

An annual sequence of wages in England starting in 1245 is used. It is shown that a standard AK-type growth model with capital externality and stochastic productivity shocks is unable to explain important features of the data. Random returns to scale are then considered. Moderate episodes of increasing returns to scale and growth are shown to be compatible with convergence of wage's process towards a unique stationary *distribution*. This holds true for other relevant values such as GDP and/or capital stock. Furthermore, random returns to scale generate heteroskedasticity, a feature common to macroeconomic time series. Finally, the limit distribution of real wages displays fat tails if returns to scale are episodically increasing. Several inference results supporting randomness of returns to scale are provided.

1. Introduction

In this paper, we make use of one of the longest available macroeconomic datasets *i.e.* an annual sequence of real wages in England starting in 1245 reported in Fig. 1 to investigate its dynamic properties. This exceptionally long sequence is available at http://www.measuringworth.org/ukearncpi/.

The most striking feature of this sequence is the dramatic increase in real wages during the XIX-th and XX-th centuries. Wage growth in the oldest part of the sample is undoubtedly smaller. Similar patterns may be obtained for other developed countries and other time series, such as GDP (see Maddison, 2000).

The second striking feature is the heteroskedasticity of this time series. In Fig. 2, we plot the increment of the log of real wages, a measure of the annual real growth rate.

The variability of the growth rate during the Late Middle Ages (1245–1500) and Modern period (1500–1800) is obviously much larger than it is in the most recent period. The high volatility of wage growth in the Middle Ages can mostly be explained by variations in nominal prices in the agricultural sector. During this period, climatic events and conflicts explain the erratic fluctuations in agricultural returns causing large fluctuations in wages. However, variations in the wage growth rate can hardly be considered as being caused by purely random short-term factors, as periods of large or low volatility seem to be quite persistent over time.

It is usual, in the literature, to decompose such time series into a trend and a transitory component. Many procedures and approaches have been proposed to obtain such decompositions, among which filtering and co-integration are currently the

* Correspondence to: Ensai, Campus de KerLann, Rue Blaise Pascal, 35172 Bruz, France. Tel.: +33 2 99 05 32 71; fax: +33 2 99 05 32 05.
E-mail address: stephane.auray@ensai.fr (S. Auray).

http://dx.doi.org/10.1016/j.csda.2014.02.011
0167-9473/

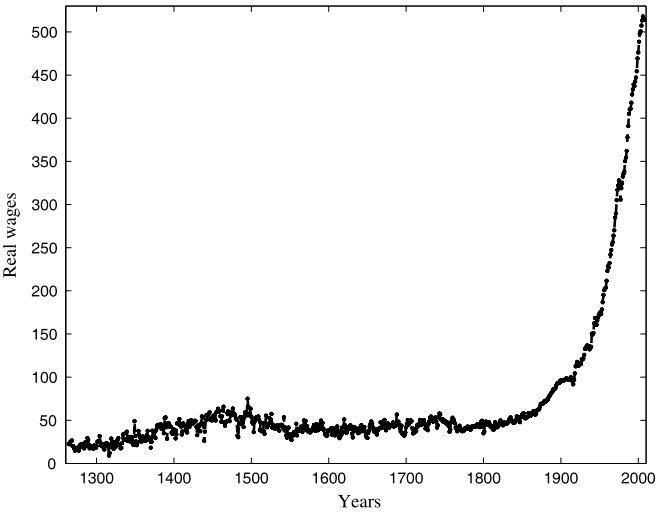

Fig. 1. Real wages.

most popular. By and large, Growth Theory focuses on the trend component, whereas transitory components are the scope of business cycle models. Variations in the trend component most often rely on separate explanations of the so-called Malthusian, Post-Malthusian and Modern Growth Regimes. The analysis of business cycles is conducted once the trend component has been removed. The current Cycle Theory views business cycles as the consequence of the transmission of exogenous shocks across sectors, GDP components, and/or countries. Noticeably, the business cycle literature almost exclusively focuses on post-war data.

Recently however, the so-called Unified Growth Theory (see Galor, 2005, 2007, Galor and Moav, 2004, Galor et al., 2009, Galor and Weil, 1999, and Galor and Weil, 2000, among many others) has proposed a single law of motion instead of using different ones to fit various historical episodes. In addition, an alternative and original approach to the understanding of business cycles has been proposed by Barro (2006), providing a middle-term perspective on economic crises. This proposal analyzes a large international panel of long time series that includes large crisis (“disasters”) and allows to analyze their impact on, among other things, the equity premium puzzle.

In this paper, we propose a structural model that allows for a joint analysis of variations in the level and the variability of growth in a long-term perspective. We eschew the decomposition into trend and transitory components, and confront the model predictions to the entire sequence of wages. As the full presentation of the model makes clear, we consider that random shocks are *always* present. Thus the relevant macroeconomic quantities will never converge towards a unique value. As a consequence, ‘long-term analysis’ refers to the study of limit distributions of relevant (random) economic quantities such as real wage, GDP and the capital stock. In what follows, we use the terms ‘stationary’ and/or ‘long term’ to refer to the limit distribution of a process.

The model is a modified version of the RBC model initially proposed by Hercowitz and Sampson (1991), that has the advantage to allow for a closed-form solution. As usual in the endogenous growth literature, an externality generated by capital accumulation generates growth. The originality of the model is to allow the strength of this externality to vary over time. Variability of returns to scale has been used to investigate the relevance of the unit root hypothesis and its consequences for growth (see Glachant, 1994 and the references therein).

We derive the optimal response of agents to Total Factor Productivity (TFP hereafter) and externality shocks. In the general case, we show that wages admit a Random-Coefficient ARMA representation, a particular case of Kesten’s process (see Kesten, 1973), if the TFP process admits a strong ARMA representation. In the special case of a constant externality on capital accumulation, the sequence of real wages admits a strong ARMA representation with fixed coefficients. The emphasis on ‘strong’ ARMA is relevant here since we cannot rely on autocorrelation structures. Indeed, in the general case, the limiting process may not be integrable in the $\mathcal{L}^2$ sense.

The model has several interesting implications. First, we provide conditions under which the real wage process admits in the limit a unique *distribution*. Under these conditions this will also be the case of other relevant aggregate economic quantities. Second, we show that agent’s optimal response to i.i.d. TFP and externality shocks generates dynamic heteroskedasticity. In the model, positive externality shocks can induce temporarily increasing returns to scale and result in recurrent transitory growth episodes. We derive analytical conditions under which these episodes are consistent with convergence of the process towards a limit distribution. This distribution exists as long as increasing returns to scale are not too frequent. Third, the stationary distribution of real wages typically displays fat tails, even if shocks are bounded. Again, this also holds true for GDP, and the capital stock. We show that fat tails are a direct consequence of episodically increasing returns to scale. Importantly, the model is plainly compatible with the vast empirical macroeconomic literature on random-walk hypothesis.