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Long memory with stochastic variance model: A recursive analysis for US inflation

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ABSTRACT

The time series characteristics of postwar US inflation have been found to vary over time. The changes are investigated in a model-based analysis where the time series of inflation is specified by a long memory autoregressive fractionally integrated moving average process with its variance modelled by a stochastic volatility process. Estimates of the parameters are obtained by a Monte Carlo maximum likelihood method. A long sample of monthly core inflation is considered in the analysis as well as subsamples of varying length. The empirical results reveal major changes in the variance, in the order of integration, in the short memory characteristics, and in the volatility of volatility. The findings provide further evidence that the time series properties of inflation are not stable over time.

1. Introduction

Monetary authorities and financial institutions require accurate measurements for inflation to assess the real value of wealth, income and returns. The demand for econometric models for inflation series is therefore high. It is well established that the statistical properties of postwar US inflation underwent a number of structural breaks. There are many possible ways to explain persistent changes in the mean, variance and autocorrelation of US inflation: technological progress; unemployment changes; output gap disturbances; fluctuations in real unit labour costs as in Galí and Gertler (1999); oil price shocks as in Hooker (2002); changes in the sectoral distribution of price changes as in Ball and Mankiw (1995). Cecchetti et al. (2007) provide an extensive discussion of the empirical evidence of changes in the time series properties of inflation. It has been recognised that empirical models should allow for such changes in mean, volatility and persistence of inflation. In particular, the Great Inflation period of the 1970s and early 1980s shows the highest mean, the highest volatility and the highest persistence in mean and volatility. The statistical significance and the economic importance of the changes in mean and persistence are debatable but the reduction in volatility since the mid 1980s is relatively undisputed.

Many econometric analyses of inflation have found evidence of long memory dynamics. The introduction of long memory features in models for price processes has started with Mandelbrot (1969); see Baillie (1996) and Robinson (2003) for econometric literature reviews. Long memory models only started to become widely used in the 1980s when Geweke and Porter-Hudak (1983) developed the log periodogram regression estimator for the order of integration parameter d in the ARFIMA model of Granger and Joyeux (1980) and Hosking (1981). They applied their estimator to postwar US CPI inflation data and found that inflation was integrated of an order d, $I(d)$, with d around 0.5, clearly different from the values of zero and one which were assumed in the earlier literature. This was an interesting alternative characterisation of the long-run behaviour of inflation with important economic implications. Moreover, the parameterisation provided simple tests

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for non-stationary values of $d > 0.5$ against stationary values $d < 0.5$ and vice versa. Baillie et al. (1996) have found evidence of long memory in international inflation rates by estimating fractional orders of integration using approximate maximum likelihood in the time domain. Their estimates took account of autoregressive conditional heteroskedasticity, initially introduced by Engle (1982) to model heteroskedasticity of UK inflation. Beran and Feng (2001) and Feng et al. (2007) developed estimation and inference methods for the ARFIMA-GARCH model in combination with semi-parametric estimates of deterministic flexible smooth trends.

A range of inflation series for different countries has provided strong evidence of high persistence and changing volatility over time. High estimates of inflation persistence in ARMA or in ARFIMA models could partly be explained by structural shifts in the unconditional mean. These shifts may lead to overestimating the persistence and to wrong conclusions regarding the order of integration of the stochastic part of the model. This point was strongly made by Perron (1989), who analysed the effect of structural changes in the mean on tests for the unit root hypothesis developed by Dickey and Fuller (1979). For example, allowing for seven breaks in the model for the mean of the US CPI index he concluded: "After 1929 the unit root is no longer present", cf. Perron (1989, p. 1385). In the context of ARFIMA models for international inflation rates Bos et al. (1999) showed how structural changes in the mean affect estimates of the fractional integration parameter d. Lobato and Savin (1998) found similar effects for stock market volatility.

If the mean is modelled as a stochastic process with occasional extremely long-lasting changes in the mean, it can be considered as a substitute for long-memory, removing the scope for an additional ARFIMA process to model deviations from this mean, see Engle and Smith (1999), Parke (1999), Diebold and Inoue (2001), and Leipus et al. (2005). Also, a fractionally integrated process can be approximated by an ARMA process with AR and MA roots close to the unit circle, see Brodsky and Hurvich (1999). When evidence of long memory is given, we may take such alternative represenations into consideration as well.

In this paper we investigate changing time series characteristics of inflation by considering different time-varying features jointly in an ARFIMA model. The heteroskedasticity is specified by means of a stochastic volatility (SV) model. SV models have been reviewed by Taylor (1994) and in the edited volume of Shephard (2005). We examine changes in the long-run persistence in the mean by monitoring changes in the estimate of the long-memory parameter d of the ARFIMA model. Furthermore, we investigate fluctuations in the volatility pattern, in the persistence of volatility and in short memory characteristics.

In contrast to Primiceri (2005) and Stock and Watson (2007) we use monthly time series data and we adopt maximum likelihood methods instead of Bayesian methods. We extend the analysis of Stock and Watson (2007) by using an ARFIMA specification which allows us to circumvent the a priori choice for the order of integration. However, as we do not use an unobserved components model we can only distinguish a single source of stochastic volatility.

We modify the Monte Carlo maximum likelihood estimation procedure of Koopman and Bos (2004) to allow its treatment within an ARFIMA model. The details are given in this paper. The evaluation of the loglikelihood function is comparatively fast, so that we are able to obtain the rolling and recursive estimates necessary for the analysis of time variation of the parameters. We label our model ARFIMA–SV. It should not be confused with the long memory SV models of Breidt et al. (1998) and Brockwell (2007) where the focus is on long memory in the variance process of the time series rather than on the time series itself.

The remainder of this paper is organised as follows. Section 2 introduces the ARFIMA-SV model and the maximum likelihood estimation of the fixed parameters and the volatility component. Section 3 presents simulation results and focuses on the precision and computational complexity of the estimating procedure. Empirical results for US inflation are given in Section 4. We present estimation results for the ARFIMA-SV model and analyse parameter (in)stability. We also use our framework to compare the results for the ARFIMA-SV model with specifications from the existing literature. Section 5 concludes.

2. Long memory model with stochastic volatility

Our model combines a long memory model as specified in ARFIMA-form by Granger and Joyeux (1980) with a stochastic volatility model as presented originally by Taylor (1982). Note that our method is not confined to ARFIMA models however, as we can consider all general linear time series models with a stationary autocorrelation function. In terms of asymptotic representations the data generating process of our approach relates to the stationary "fractional Brownian motion of Type I", see the discussion in Davidson and Hashimzade (2008) for the difference with the (nonstationary) Type II fractional Brownian motions in the development of asymptotic theory.

We choose the ARFIMA specification as it delivers the well known ARIMA models as special cases. Moreover, the ARIMA(0, 1, 1) model encompasses the random walk plus noise model, or local level model, which was found to be very effective for forecasting inflation, see e.g. Stock and Watson (2007). Sowell (1992) discussed the exact maximum likelihood estimation of ARFIMA parameters for stationary data. Doornik and Ooms (2003) refined Sowell's method and introduced a computationally efficient implementation. Exact maximum likelihood avoids ad hoc decisions on the likelihood for the initial observations of the process. Because of the relatively strong correlation of distant observations this is more important for long memory ARFIMA models than for short memory ARMA models.

We add the SV specification to the ARFIMA model. The SV specification avoids the ad-hoc decisions for the computation of the likelihood of the initial observations that one needs to make in estimating GARCH type models as introduced by