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Estimating GARCH-type models with symmetric stable innovations: Indirect inference versus maximum likelihood

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ABSTRACT

Financial returns exhibit conditional heteroscedasticity, asymmetric responses of their volatility to negative and positive returns (leverage effects) and fat tails. The α-stable distribution is a natural candidate for capturing the tail-thickness of the conditional distribution of financial returns, while the GARCH-type models are very popular in depicting the conditional heteroscedasticity and leverage effects. However, practical implementation of α-stable distribution in finance applications has been limited by its estimation difficulties. The performance of the indirect inference approach using GARCH models with Student's t distributed errors as auxiliary models is compared to the maximum likelihood approach for estimating GARCH-type models with symmetric α-stable innovations. It is shown that the expected efficiency gains of the maximum likelihood approach come at high computational costs compared to the indirect inference method.

1. Introduction

Most of the financial returns exhibit conditional heteroscedasticity and heavy-tailedness. While conditional heteroscedasticity is standardly captured by means of GARCH or stochastic volatility (SV) models (e.g. Bollerslev (1986) and Ghysels et al. (1996)), depicting the empirically observed fat-thickness of financial returns is not always straightforward. Although theoretically most of the GARCH and SV specifications can accommodate for fat-tailedness through their specification, in practice, in most of the cases, there is still excess kurtosis left in the standardized residuals. A very common solution to this problem is to assume a fat-tailed distribution for the standardized innovations of the conditional heteroscedasticity models, and the Student's t is a natural candidate (e.g., Calzolari et al. (2003)). However, one drawback of the Student's t distribution is that it lacks in stability under aggregation, which is of particular importance in portfolio applications and risk management. A fat-tailed distribution that overcomes the drawbacks of the Student's t is the α-stable. Its theoretical foundations lay on the generalized central limit theorem. Moreover, similar to the Student's t distribution, the α-stable can be easily adapted to account for asymmetry in the underlying series. The main drawback of this specification is its estimation. The fact that, for most of the parameters constellations, the α-stable does not have a closed-form density specification or the theoretical moments simply do not exist, makes the estimation of its parameters a cumbersome task and limits the interest among academics and practitioners. Proposals of likelihood-free inference are only recently available in the Bayesian context: e.g., Peters et al. (2012).

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In this paper we focus on estimating GARCH-type models with symmetric α-stable innovations by means of the indirect inference (IndInf) method proposed by Gouriéroux et al. (1993) and maximum likelihood (ML) as described in Nolan (1997). The indirect inference estimation approach has already proved its adequacy in estimating the parameters of the stable distribution in Lombardi and Calzolari (2008) and Lombardi and Veredas (2009) and Garcia et al. (2011). Lombardi and Calzolari (2009) use the indirect inference approach to estimate a SV model with α-stable innovations. Differently from their approach, we focus on comparing the estimation results stemming from IndInf and ML when estimating GARCH-type models with symmetric α-stable innovations. We focus on estimating GARCH-type specifications due to their popularity among practitioners and academics. The popularity of GARCH models over SV models originates in their ability of straightforwardly capturing empirical features of financial volatilities, such as: asymmetric responses to negative and positive returns, known in literature as leverage effects, high persistence or long memory as well as causality and correlation effects with further economic variables such as: Volatility Index (VIX), inflation, etc.. To illustrate this, we estimate, besides simple GARCH specifications, also Threshold GARCH (TGARCH) models as introduced by Glosten et al. (1993) that capture leverage effects, which are highly relevant in financial applications.

In the GARCH context, the α-stable distribution is first mentioned by de Vries (1991) and Ghose and Kroner (1995), while the GARCH model with α-stable innovations is first proposed by McCulloch (1985) within a restricted framework and by Liu and Brorsen (1995b) within a more general context. The theoretical stationarity properties of GARCH models with α-stable innovations are studied by Panorska et al. (1995) and Mittnik et al. (2000, 2002). In what regards the estimation, Liu and Brorsen (1995a) propose the ML approach, however for very specific values of the parameters and GARCH specifications.

The aim of our paper is to alleviate the estimation problems in implementing GARCH-type models with symmetric α-stable innovations under a very general parameter setting. Our implementation does not impose any parameter or model specification constraints. The IndInf estimation method uses GARCH-type specifications with Student's innovations as auxiliary models. The choice of the auxiliary model is motivated by the fact that there is a rather natural correspondence between the two models: besides having the same number of parameters and common GARCH-type specifications for the conditional heteroscedasticity, the degrees of freedom in the Student's t distribution is the direct counterpart of the parameter of stability or characteristic exponent in the stable distribution, as both measure the tail-thickness of the distribution. In what regards the ML implementation, we apply the method described by Nolan (1997) and Matsui and Takemura (2006) based on the numerical evaluation of the symmetric stable density and its derivatives for a wide range of parameter constellations. Furthermore, we adapt their procedure to estimate both the parameters of the symmetric α-stable distribution and of the GARCH specifications. As an alternative to Nolan's (1997) approach, one can consider the approach of Chenyao et al. (1999) that uses fast Fourier transforms to approximate the stable density functions.

Thus, our paper, besides contributing to the existing literature for implementing further types of GARCH models, such as TGARCH models with symmetric α-stable innovations, it also directly compares the performance of standard ML and IndInf when estimating a wide range of GARCH-type models with symmetric α-stable innovations.

Within a thorough Monte Carlo experiment and an empirical application to twelve time series of financial returns of DJIA, SP500, IBM and GE, sampled at different frequencies (daily, weekly, monthly), we provide valuable empirical evidence in favor of applying IndInf over ML under a very general model specification and parameter settings. We show that, although both methods provide accurate estimates, the expected efficiency gain of ML comes at high computational costs: besides being easier to implement, IndInf reports estimation results up to ten times faster than ML.

The rest of the paper is organized as follows: Section 2 gives a short introduction to the symmetric α-stable distributions, Section 3 focuses on describing the models of interest, namely GARCH and TGARCH with symmetric α-stable innovations, Section 4 shortly introduces the estimation methods and describes their practical implementation for estimating the models of interest. Section 5 presents the results of a Monte Carlo experiment, while Section 6 shows results from estimating the models on real data. Section 7 concludes.

2. Symmetric α-stable distributions

The stable family of distributions, which is also known under the name α-stable, constitutes a generalization of the Gaussian distribution by allowing for asymmetry and heavy tails. In this paper, we focus on the symmetric stable distribution, which is a subclass of the stable family of distributions with no asymmetry. From a theoretical point of view, the use of models based on stable distributions is justified by the generalized version of the central limit theorem, in which the condition of finite variance is replaced by a much less restricting one concerning a regular behavior of the tails. It turns out that stable distributions are the only possible limiting laws for normalized sums of iid random variables (Feller, 1966). The lack of closed formulas for density and distribution functions (except for a few particular cases) has been, however, a major drawback of the stable distributions in applied fields.

In general a random variable X is said to have a stable distribution if and only if, for any positive numbers c_1 and c_2, there exists a positive number k and a real number d such that

$$kX + d \stackrel{d}{=} c_1X_1 + c_2X_2, \tag{1}$$

where X_1 and X_2 are independent and have the same distribution as X and $\stackrel{d}{=}$ stands for equality in distribution. If $d = 0$, X is said to be strictly stable. In order to show that the stable distribution is a generalization of the normal, let the variable