Computational Statistics and Data Analysis ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at SciVerse ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

When long memory meets the Kalman filter: A comparative study

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ARTICLE INFO

Article history:
Received 22 November 2011
Received in revised form 28 September 2012
Accepted 26 October 2012
Available online xxxx

Keywords:
ARFIMA models
State space
Missing observations
Measurement error
Level shifts

ABSTRACT

The finite sample properties of the state space methods applied to long memory time series are analyzed through Monte Carlo simulations. The state space setup allows to introduce a novel modeling approach in the long memory framework, which directly tackles measurement errors and random level shifts. Missing values and several alternative sources of misspecification are also considered. It emerges that the state space methodology provides a valuable alternative for the estimation of the long memory models, under different data generating processes, which are common in financial and economic series. Two empirical applications highlight the practical usefulness of the proposed state space methods.

1. Introduction

Long-range dependent data arise in a wide variety of scientific disciplines, from hydrology to economics. The subject of long memory time series was brought to prominence by Hurst (1951) on hydrological time series and has subsequently received extensive attention in the statistical literature. The use of fractional processes in economics and econometrics has been introduced by the seminal paper by Granger (1980) and Granger and Joyeux (1980). The volumes by Beran (1994) and Palma (2007) and the collection of Robinson (2003) provide detailed introductions to the statistical and econometric analysis of the long memory time series. The starting point of the econometric literature on autoregressive fractionally integrated moving average (ARFIMA henceforth) models has been motivated by the fact that many economic and financial time series show evidence of being neither $I(0)$ nor $I(1)$. Nowadays, a broad range of applications in finance and macroeconomics shows that fractional integration and long memory are relevant, see among others Diebold et al. (1991) for exchange rate data, Andersen et al. (2001a,b) for financial volatility series, and Baillie et al. (1996) for inflation data. Early papers on the estimation of the long-range dependent models are Fox and Taqqu (1986), Dahlhaus (1989), Sowell (1992) and Robinson (1995). See Chan and Palma (2006) for a more recent review.

In an alternative framework, Chan and Palma (1998) proposed a state space approach to compute the maximum likelihood (ML henceforth) estimates of the ARFIMA parameters. The authors suggest to truncate the infinite MA and AR representations of an ARFIMA model and to write the ARFIMA in state space form. The long memory parameter, d, can be then estimated by means of the Kalman filter (*KF* henceforth) recursions. The estimates obtained by this method are consistent, asymptotically normal and efficient under mild regularity conditions. This methodology, although conceptually simple, was computationally very intensive in the 90's, and, for this reason, not commonly used. With the computational capabilities at hand nowadays, it is possible to estimate such models in a few seconds even for large datasets. Several simulation studies, by Bisaglia and Guegan (1998), Nielsen and Frederiksen (2005), Haldrup and Nielsen (2007) and Rea et al. (forthcoming) provide a comparison between different estimation strategies, but without considering the state space alternative. This underlines

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doi:10.1016/j.csda.2012.10.018

Please cite this article in press as: Grassi, S., Santucci de Magistris, P., When long memory meets the Kalman filter: A comparative study. Computational Statistics and Data Analysis (2013), doi:10.1016/j.csda.2012.10.018

that, although, great effort has been spent in the estimation of fractional processes with semiparametric and maximum likelihood methods, little has been done to explore the state space alternative.

This paper aims at filling this gap through an extensive Monte Carlo simulation exercise. The main contribution is the assessment of the practical usefulness of the state space approach in dealing with potential contaminations of the long memory signal. The Monte Carlo simulations are therefore intended as an attempt to explore the potentiality of the state space approach in the long memory framework. With this purpose, several parametric and semiparametric estimation methods for the ARFIMA models are considered and compared with the state space alternatives. The various estimation methods are contrasted using several data generating processes with alternative short-run dynamics including the possibility of t-distributed and Skew Normal innovations, missing values, measurement errors and level shifts. It is shown that in scenarios, relevant for practical purposes, in which the long memory signal is contaminated by different sources of noise, such as measurement errors and level shifts, the state space approach outperforms the traditional estimators. The Monte Carlo simulations highlight the reliability of the state space methods also in small samples, due to the straightforward modeling of measurement errors, level shifts and missing values, which allows to filter the long memory signal. It should be stressed that the use of the state space approach to account for measurement errors and level shifts is a novel contribution in the long memory framework.

The main results can be summarized as follows: firstly, the state space methodology is a valid alternative to the usual estimation procedures in standard cases, i.e. when the long memory signal is not contaminated by any additional feature. However, as a parametric approach, it suffers from the possible misspecification of the ARMA dynamics. Secondly, the estimates based on the state space approach have good finite sample performances also when the long memory innovations are non-Gaussian, with non-zero skewness and high kurtosis. Thirdly, when the series at hand contains missing observations, then the state space estimation method is superior to the traditional ones as it has low bias and RMSE. Fourthly, in case of measurement errors, the *KF* largely outperforms the traditional estimators as well as the corrected local Whittle of Hurvich et al. (2005). Finally, a novel and promising approach to the joint modeling of long memory and level shifts is provided by a modified version of the Lu and Perron (2010) filter, that can handle both features at the same time. Differently from usual estimators, that are upwardly biased, the proposed method produces unbiased estimates of the d parameter and a good coverage at 95% confidence, even in presence of level shifts.

The paper is organized as follows. The class of ARFIMA models is introduced in Section 2. The state space methodology is presented in Section 3. Section 4 reports the extensive Monte Carlo experiments and the methodological contributions that are the core of this paper. Section 5 provides two empirical applications of the long memory state space models. Section 6 concludes the paper.

2. ARFIMA processes

An ARFIMA(p, d, q) process y_t is defined as:

$$\Phi(L)(1-L)^d y_t = \Theta(L)\eta_t, \tag{1}$$

where $\{\eta_t\}$ is a sequence of independent random variables with zero mean and constant variance σ_η^2, the lag operator L is such that $Ly_t = y_{t-1}$; $\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$ is the autoregressive polynomial, while $\Theta(L) = 1 + \theta_1 L + \cdots + \theta_q L^q$, is the moving average operator, such that $\Phi(L)$ and $\Theta(L)$ have all their roots outside the unit circle, with no common factors. The long memory property is induced by the term $(1-L)^d$, which is the fractional difference operator. The parameter d determines the long memory degree of the process. If $d > -1/2$ the process is invertible and possesses a linear representation. If $d < 1/2$ it is covariance stationary. Furthermore, if $d > 0$ the process is said to have long memory since the autocorrelations die out at an hyperbolic rate (and indeed are no longer absolutely summable) in contrast to the much faster exponential rate in the weak dependence case. If $d = 0$ the spectral density is bounded at the origin and the process is an ARMA with only weak dependence (short memory). The fractional difference operator $\Delta^d = (1-L)^d$ in Eq. (1) is defined by its binomial expansion:

$$(1-L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)} L^j, \tag{2}$$

where $\Gamma(\cdot)$ is the Gamma function. Hosking (1981) shows that a stationary ARFIMA(p, d, q) admits infinite MA and AR expansions as

$$y_t = \sum_{j=0}^{\infty} \psi_j \eta_{t-j}, \tag{3}$$

$$y_t = \sum_{j=1}^{\infty} \pi_j y_{t-j} + \eta_t. \tag{4}$$

Hosking (1981) also provides a formula to compute the weights ψ_j and π_j for low order ARFIMA processes, which are obtained by an infinite convolution of the ARMA terms with the fractional difference operator. An alternative, although not