Computational Statistics and Data Analysis ■ (■■■■) ■■■–■■■

Contents lists available at ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

Estimation of risk measures in energy portfolios using modern copula techniques

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ARTICLE INFO

Article history:
Received 1 March 2013
Received in revised form 23 January 2014
Accepted 25 January 2014
Available online xxxx

Keywords:
Crude oil
Joint dynamics
Tail copulas
Goodness-of-fit tests
Backtesting
Risk measures

ABSTRACT

The dependence structure between WTI and Brent crude oil spot log-returns is analysed using modern copula techniques. In a first step, to account for autocorrelation and volatility clustering in the marginals, several single equation models are applied. Second, to select both copulas and tail copulas characterising the joint dynamics between the time series, newly introduced bootstrap-based goodness-of-fit tests are implemented and evaluated. Based on each approach, a comprehensive backtesting is performed by simulating and comparing the risk measures Value-at-Risk and Expected Shortfall with observed values.

1. Introduction

The modelling of stochastic dependence in energy and commodity markets via copulas has become increasingly common in applications. Especially crude oil, which plays a major role in commodity investments, has been related to other markets (see, for instance Reboredo (2012), Wu et al. (2012) and Aloui et al. (2013) for foreign exchange rate markets or Wen et al. (2012) for the stock market). When staying within the energy sector Grégoire et al. (2008) analyse the dependence structure of log-returns of futures on crude oil and natural gas, Accioly and Aiube (2008) study the co-movement of crude oil and gasoline prices, while Reboredo (2011) focuses on the dependence structure between crude oil benchmark prices. Using weekly data the article examines whether crude oil markets are rather globalised or regionalised. Having estimated each log-return time series individually, the author accounts for the dependence between the different crude oil grades by fitting various copula families to the error terms. The unknown parameters of the copula functions are obtained via maximum likelihood, whereas the decision which model performs best relies on an adjusted information criterion and a pseudo-likelihood ratio test. Additionally and directly linked to the estimated copula parameters, coefficients for upper and lower tail dependence are provided.

According to Jäschke et al. (2012) a general goodness-of-fit test for copulas does not necessarily provide a good model of tail dependence, as most of these procedures take the whole support of the distribution into account and therefore, adopting copula inference techniques for modelling joint extreme events can be very misleading for risk management purposes. Using the concept of tail copulas accounts for all possible scenarios of joint extreme outcomes and thus decouples the decision to select an appropriate model describing the overall dependence from the analysis of the joint dynamics in the tail of the underlying distribution.

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http://dx.doi.org/10.1016/j.csda.2014.01.019
0167-9473/

The present paper follows this route and extends the current state of knowledge within a couple of areas. First and foremost, we apply the partial derivatives multiplier bootstrap goodness-of-fit test for tail copulas (Bücher and Dette, 2013) to the log-returns of two crude oil grades. To better compare the findings with a traditional copula fit we present both the best model characterising the overall dependence structure using a bootstrap-based goodness-of-fit test for copulas and the newly introduced Copula Information Criterion (Grønneberg and Hjort, 2014). Moreover, from a risk management perspective it is indispensable to capture the joint behaviour of certain assets within energy portfolios. Following Jäschke et al. (2012), a thorough understanding of energy portfolio risk requires an adequate assessment of the probability that large negative co-movements occur together, i.e. lower tail dependence. Therefore, the present paper introduces a wide and comprehensive backtesting framework for two of the most commonly applied risk measures, namely Value-at-Risk and Expected Shortfall.

Although the problem of modelling and assessing risk measures in energy portfolios using copulas is not completely new (see Liu, 2011; Lu et al., 2011; Wu et al., 2012), the backtesting approach derived from the techniques described above, to the best of our knowledge, has not yet been addressed before. The paper's claim to cover a topic of broad applicability and high practical relevance is backed by extensive guidelines from a practitioner's point of view as well as detailed comments concerning the empirical implementation.

The present analysis uses a large data set of daily quotes of WTI Cushing Crude Oil Spot and the Bloomberg European Dated Brent. Quotes of WTI are commonly used as a reference spot price for US crude oil whereas the price for Brent serves as a benchmark for European crude oil. Both oil grades belong to the class of light and sweet crude oils, i.e. they are characterised by their low density and their low sulphur content. Although Brent is not as light or as sweet as WTI, it is still high-grade for subsequent processing and thus, both crude oils are not completely, but to a great extent, generally treated as interchangeable.

Due to these differences in quality, WTI futures were usually traded at a small premium to Brent futures. From early on in 2011 the development of the spread between the two futures has attracted the attention of energy and finance markets. Analysts and investors have become quite uncertain how to play the WTI–Brent anomaly. In fact, Brent did not only trade over WTI, but also the spread of the futures widened notably, which is reason enough to analyse the dependence structure between the two crude oil grades from a probabilistic point of view.

Historically, market participants often have not only assumed a physical interchangeability but accordingly a portfolio-interchangeability. Thus one could have thought of an almost perfect correlation between the two oil grades. After all, in view of the WTI–Brent anomaly, risk management needs to address the following two points. First, if a portfolio contains only one of the two varieties, it is really important to precisely consider and simulate the kind one is holding. Second, in the context of risk optimisation with fixed expected returns it can be analysed, whether or not it makes sense to hold both types simultaneously and thus diversifying the portfolio. However, should one keep both, a dedicated analysis of dependence is crucial since apparently structures have changed significantly and a simple correlation approach may mislead.

The paper is organised as follows: Section 2 briefly introduces the well-known copula framework and reviews the closely related concept of tail copulas with their corresponding non-parametric estimates. Section 3 models the individual time series of both crude oil grades, accounting for serial dependence in the data. Section 4 describes the applied rank-based goodness-of-fit tests for copulas and tail copulas, respectively. Section 5 provides instruction on how to put the described theory into practice. Section 6 backtests the findings using the example of risk measures. Finally, Section 7 summarises the results and the used methods.

2. Preliminaries

2.1. Copulas and tail dependence

The theory of copulas investigates the dependence structure of multivariate distribution functions. As this article focuses on co-movements between the WTI and Brent crude oil futures, we state all further definitions and results for the bivariate case only. From a probabilistic perspective, a copula is a joint distribution function with uniformly distributed margins on the interval $[0, 1]$. To begin with, we consider a random vector (X, Y) with continuous marginal distribution functions $F(x) := \mathbb{P}[X \leq x]$ and $G(y) := \mathbb{P}[Y \leq y]$, $x, y \in \mathbb{R}$, respectively. The theoretical foundation for the application of copulas is provided by Sklar's theorem (Sklar, 1959), according to which there exists a unique copula C, called the copula of X and Y, such that

$$\mathbb{P}[X \leq x, Y \leq y] = C(F(x), G(y)), \tag{1}$$

for all $x, y \in \mathbb{R}$. Conversely, if C is a copula and F and G are distribution functions, then the function defined by Eq. (1) is a bivariate distribution function with margins F and G, respectively. It can be shown that

$$C(u, v) = \mathbb{P}[U \leq u, V \leq v] = \mathbb{P}[X \leq F^{(-1)}(u), Y \leq G^{(-1)}(v)], \tag{2}$$

for all $u, v \in [0, 1]$, where $F^{(-1)}$ and $G^{(-1)}$ denote the quasi-inverses of F and G, respectively, i.e.

$$F^{(-1)}(u) = \inf\{x \in \mathbb{R} \mid F(x) \geq u\}, \tag{3}$$

for all $u \in [0, 1]$ (analogously for G). Summing up, copulas allow for a step-wise modelling of the joint distribution whose dependence structure is independent of the respective marginal distributions. Further details can be found in Nelsen (2006).

Please cite this article in press as: Jäschke, S., Estimation of risk measures in energy portfolios using modern copula techniques. Computational Statistics and Data Analysis (2014), http://dx.doi.org/10.1016/j.csda.2014.01.019