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Panel cointegration testing in the presence of a time trend

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ABSTRACT

A new likelihood-based panel cointegration test which allows a linear time trend in the data generating process is proposed. The test is an extension of the likelihood ratio type test with trend adjustment prior to testing to the panel data framework. Under the null hypothesis, the standardized statistic has a limiting normal distribution as the number of time periods and the number of cross-sections tend to infinity sequentially. Additionally, an approximation involving the moments based on a vector autoregressive process of order one is introduced. A Monte Carlo study demonstrates that the test has reasonable size and high power in finite samples.

1. Introduction

In the last two decades the cointegration methodology has become popular to analyze long-run stationary relationships among integrated economic variables. The performance of a cointegration test can be improved by its extension to the panel data framework which enlarges the underlying data basis. Most macroeconomic variables, e.g. prices, gross domestic product, consumption etc., exhibit a trending behavior. To model this behavior in the multivariate time series literature a drift parameter is included in the vector autoregressive (VAR) model. Building on this idea, Saikkonen and Lütkepohl (2000) proposed Lagrange multiplier (LM) and likelihood ratio (LR) type cointegration tests for data with a linear time trend which are different from the popular Johansen (1995) test. Saikkonen and Lütkepohl (2000) based their tests on the idea of subtracting estimates of the deterministic terms from the original data and applying the cointegration test on the trend-adjusted data. The principle of subtracting estimates of the deterministic terms of the model was first suggested by Stock and Watson (1988). Saikkonen and Lütkepohl (2000) proposed to estimate the deterministic terms under the null hypothesis using a generalized least squares (GLS) method. By construction, under the null hypothesis the limit distribution of their tests does not depend on the parameter values of the deterministic terms (see Lütkepohl et al., 2001, who pointed out that the test statistics are even invariant to the choice of the deterministic terms). In a simulation study, they concluded that their tests have better properties than the test of Johansen (1995) allowing for a linear trend. Moreover, the LR type version of their tests outperforms the LM type version.

So far there are only a few examples of likelihood-based panel cointegration tests which allow for a deterministic linear trend in the data generating process (DGP). Larsson et al. (2001) who extended the Johansen trace test to panel data and Breitung (2005) who based his tests on the procedure of Saikkonen (1999), mentioned in their papers that their panel cointegration tests can be extended to the case with deterministic terms, but they did not deliver any proof of corresponding asymptotic results. Additionally, Anderson et al. (2006) introduced a systems panel cointegration test, which allows for a

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linear time trend. This test is built on the method of Box and Tiao (1977) in which the number of stochastic common trends is determined by the number of certain eigenvalues close to one. Note that these eigenvalues are the squared canonical correlation coefficients between a multivariate time series and its linear projection on its own history. However, there is no likelihood-based panel cointegration test that relies on the idea of subtracting the estimated deterministic terms prior to testing for cointegration.

The goal of this paper is to close this gap. We extend the trend-adjusting procedure of Saikkonen and Lütkepohl (2000) to the panel data framework and propose an LR type panel cointegration test in the presence of a linear time trend in the DGP; recall that the LR type test was superior to the LM type version in the simulation study of Saikkonen and Lütkepohl (2000). With this new likelihood-based panel cointegration test statistic one can determine the number of cointegrating relations in the system. This is advantageous compared to the single-equation tests, which can only be used to decide whether there is a cointegrating relation or not. The proposed panel SL test statistic is a standardized version of the average of the individual LR type test statistics (trace statistics) over the cross-sections. The standardization is based on the first two moments of the asymptotic trace statistic, i.e. of the limit distribution of the trace statistic. Alternatively, according to Larsson (1999) and Larsson et al. (2001) moments from an approximating VAR(1) process could be used. Under the null hypothesis, the panel SL test statistic converges in distribution to the standard normal law as the number of time periods and the number of cross-sections tend to infinity in a sequential way. Therefore standard normal quantiles may serve as critical values. To justify our approach, we show that the first two moments of the asymptotic trace statistic exist and may be obtained as limits of the moments of a certain statistic, which is used to approximate the asymptotic moments by simulation. This crucial result is an extension of a result of Karaman Örsal and Droge (2011) who corrected a related proof in Larsson et al. (2001) for the case without deterministic terms. The results of a simulation study suggest that the panel SL test has reasonable finite sample properties.

The paper is organized as follows. In Section 2 the heterogeneous panel vector error correction (VEC) model with linear time trend is introduced. Section 3 presents the new LR type panel cointegration test. The size and size-adjusted power properties are examined by means of a Monte Carlo study in Section 4. Finally, Section 5 discusses possible extensions and gives a summary of the main results. All proofs are deferred to the Appendix.

2. The model

Consider a panel data set consisting of N cross-sections (individuals) observed over T time periods and suppose that for each individual i ($i = 1, \dots, N$) a K-dimensional time series $y_{it} = (y_{1it}, \dots, y_{Kit})'$, $t = 1, \dots, T$, is observed which is generated by the following heterogeneous VAR(p_i) model with linear trend:

$$y_{it} = \mu_{0i} + \mu_{1i}t + x_{it}, \quad i = 1, \dots, N;\ t = 1, \dots, T, \tag{1}$$

$$x_{it} = A_{i1}x_{i,t-1} + \cdots + A_{i,p_i}x_{i,t-p_i} + \varepsilon_{it}. \tag{2}$$

Here μ_{0i} and μ_{1i} are unknown K-dimensional parameter vectors, p_i is the lag order of the VAR process for the ith cross-section and $A_{i1}, \dots, A_{i,p_i}$ are unknown $(K \times K)$ coefficient matrices. Moreover, we assume that the K-dimensional random errors ε_{it} are serially and cross-sectionally independent with $\varepsilon_{it} \sim N_K(0, \Omega_i)$ for some nonrandom positive definite matrix Ω_i. For simplicity the initial value condition $x_{it} = 0$, for $t \le 0$ and $i = 1, \dots, N$, is imposed. However, the results remain valid if we assume that the initial values are drawn from a fixed probability distribution, which does not depend on the sample size.

By subtracting $x_{i,t-1}$ from both sides of (2) and rearranging terms we get the VEC form of the model for x_{it}:

$$\Delta x_{it} = \Pi_i x_{i,t-1} + \sum_{j=1}^{p_i-1} \Gamma_{ij}\Delta x_{i,t-j} + \varepsilon_{it}, \quad i = 1, \dots, N;\ t = 1, \dots, T, \tag{3}$$

where $\Pi_i = -(I_K - A_{i1} - \cdots - A_{i,p_i})$ and $\Gamma_{ij} = -(A_{i,j+1} + \cdots + A_{i,p_i})$ for $j = 1, \dots, p_i - 1$. The components of the process x_{it} are assumed to be integrated at most of order one and cointegrated with cointegrating rank $r_i = \operatorname{rank}(\Pi_i)$, $0 \le r_i \le K$. In other words, y_{it} is at most $I(1)$ and cointegrated of order r_i. Thus, the matrix Π_i can be decomposed as

$$\Pi_i = \alpha_i\beta_i', \quad i = 1, \dots, N, \tag{4}$$

where both α_i and β_i are $(K \times r_i)$ matrices of full column rank. Note that α_i and β_i are the matrices of the loading coefficients and the cointegrating vectors, respectively.

On account of (1)–(3) we obtain the VEC form of y_{it}:

$$\Delta y_{it} = \nu_i + \alpha_i\left[\beta_i' y_{i,t-1} - \tau_i(t-1)\right] + \sum_{j=1}^{p_i-1} \Gamma_{ij}\Delta y_{i,t-j} + \varepsilon_{it}, \quad i = 1, \dots, N;\ t = p_i + 1,\ p_i + 2, \dots, T, \tag{5}$$

with $\nu_i = -\Pi_i\mu_{0i} + (I_K - \Gamma_{i1} - \cdots - \Gamma_{i,p_i-1})\mu_{1i}$ and $\tau_i = \beta_i'\mu_{1i}$.

To determine the number of cointegrating relations among the components of the process y_{it}, the rank of the matrix Π_i should be tested. With $\bar{r} = \max\{r_i :\ 1 \le i \le N\}$, the relevant null and alternative hypotheses for the cointegration tests are

$$H_0 : \bar{r} \le r \quad \text{versus} \quad H_1 : r < \bar{r} \le K. \tag{6}$$

That is, under the null hypothesis all the cross-sections have at most cointegrating rank r.