Computational Statistics and Data Analysis ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Computational Statistics and Data Analysis

journal homepage: www.elsevier.com/locate/csda

The indirect continuous-GMM estimation☆

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ARTICLE INFO

Article history:
Received 21 February 2012
Received in revised form 29 July 2013
Accepted 24 September 2013
Available online xxxx

Keywords:
Conditional moment restriction
Continuum of moment conditions
Covariance operator
Empirical characteristic function
Generalized method of moments
Indirect estimation

ABSTRACT

A *curse of dimensionality* arises when using the Continuum-GMM procedure to estimate large dimensional models. Two solutions are proposed, both of which convert the high dimensional model into a continuum of reduced information sets. Under certain regularity conditions, each reduced information set can be used to produce a consistent estimator of the parameter of interest. An indirect CGMM estimator is obtained by optimally aggregating all such consistent estimators. The simulation results suggest that the indirect CGMM procedure makes an efficient use of the information content of moment restrictions.

1. Introduction

In the financial econometrics literature, many models are specified directly in terms of their characteristic function (CF) because their densities are unknown. Typical examples include the stable distributions and discretely sampled continuous time processes. A discrete sample from a square root diffusion is an exception (Singleton, 2001). Its transition density is of the same form as for an autoregressive Gamma model, being an infinite mixture of Gamma densities with Poisson weights (see Gourieroux and Jasiak, 2005). Unfortunately, infinite mixtures have to be truncated in practice for the sake of feasibility. The challenge raised by the estimation of such models has motivated the use of various instances of the CF based GMM. See e.g. Carrasco and Florens (2000), Jiang and Knight (2002), Knight and Yu (2002), Yu (2004), Taufer et al. (2011) and Li et al. (2012).

In fact, two random variables have the same distribution if and only if their CFs coincide on their whole domain. This suggests that an inference procedure that adequately exploits the information content of the CF has the potential to be as efficient as a likelihood-based approach. The continuum-GMM (CGMM) proposed by Carrasco and Florens (2000) permits an efficient use of the whole continuum of moment conditions obtained by taking the difference between the theoretical CF of an IID random variable and its empirical counterpart. Carrasco et al. (2007a) extend the use of the CGMM to Markov and weakly dependent models. However, the scope of the CGMM goes beyond models specified in terms of their CF. To see this, let us consider an economic model summarized by a conditional moment restriction (CMR) of the type $E\left(g(\theta_0, y_t)|X_t\right) = 0$, where $y_t \in \mathbb{R}$, $X_t \in \mathbb{R}^d$ and θ_0 is the parameter of interest. This type of CMR is quite prevalent in the macroeconomic and asset pricing literature (e.g., first order conditions of DSGEs or Euler Equations) and it is equivalent to the infinite set of unconditional moment restrictions given by: $E\left(g(\theta_0, y_t)A(X_t)\right) = 0$ for all possible functions $A(X_t)$. Dominguez and Lobato (2004) show that using a small number of unconditional moment restrictions selected from the previous infinite set may not warrant the identification of θ_0. They show that identification and GMM-efficiency are achieved by

☆ The Matlab codes used to compute the results of this paper are available as supplementary material.

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0167-9473/$ – see front matter
http://dx.doi.org/10.1016/j.csda.2013.09.023

exploiting the continuum of unconditional moment restrictions given by $E\left(g(\theta_0, y_t)1(X_t < \tau)\right) = 0,\ \tau \in \mathbb{R}^d$. Bierens (1982) showed that the CMR $E\left(g(\theta_0, y_t)|X_t\right) = 0$ is also equivalent to the continuum of unconditional moment restrictions $E\left(g(\theta_0, y_t)\exp(i\tau' X_t)\right) = 0,\ \tau \in \mathbb{R}^d$. The latter continuum shares some similarities with the one used to design the CGMM and it has also been used by Lavergne and Patilea (2013) to derive smoothed minimum distance estimators.

The CGMM builds on the same philosophy as the GMM of Hansen (1982). Both are based on the minimization of a quadratic form associated with some scalar product. The scalar product of the GMM is defined on a finite dimensional vector space while the one used to design the CGMM is defined on an infinite dimensional Hilbert space. An example of scalar product between two functions $h(\tau)$ and $g(\tau)$ in a complex Hilbert space is given by the integral of $h(\tau)\overline{g(\tau)}$ against a continuous measure $\Pi(\tau)d\tau$, where $\overline{g(\tau)}$ is the complex conjugate of $g(\tau)$. The norm of $h(\tau)$ associated with this scalar product is given by the integral of $h(\tau)\overline{h(\tau)}$ against $\Pi(\tau)d\tau$. Hence the multiplicity of the integral is determined by the dimensionality of τ. In Carrasco and Florens (2000), $h(\tau) \equiv h(\tau, \theta_0)$ is the difference between the theoretical CF of a random variable and its empirical counterpart where τ the Fourier transformation index and θ_0 a finite dimensional parameter.

Typically, one chooses $\Pi(\tau)$ to be a multivariate Gaussian measure on $\mathbb{R}^d$ so as to be able to use Gauss–Hermite quadratures. This approach produces satisfactory results when the index τ is either one-dimensional or two-dimensional (Kotchoni, 2012); however, the complexity of the numerical integration grows exponentially with the dimensionality of τ. For instance, if 10 quadrature points deliver a given precision in numerically evaluating a one dimensional integral, approximately 10^d quadrature points would be required to obtain an equivalent precision for a d-dimensional integral. This "*curse of dimensionality*" is well-known in computational fields. Possible solutions to address this problem include reducing the number of quadrature points or removing quadrature points that have very low weights. Unfortunately, neither of these solutions provides a substantial numerical efficiency gain without jeopardizing the accuracy of the overall estimation procedure.

In an effort to circumvent the aforementioned problem, two approaches are explored in this paper. The first approach consists of converting the multivariate model into a continuum of univariate models. To begin, one draws a vector τ from a continuous distribution defined on a standardized subset Λ of $\mathbb{R}^d$ (e.g. the unit sphere or the unit hypercube). Next, one defines the set of all moment functions along the vector line τ as $h_{\tau,t}(u, \theta_0) \equiv h_t(u\tau, \theta_0),\ u \in \mathbb{R}$. Under certain regularity conditions, a consistent estimator $\widehat{\theta}_{\text{CGMM}}(\tau)$ is obtained by minimizing a norm of $h_{\tau,t}(u, \theta_0)$, which is a function of a one dimensional index u for a given τ. A final estimator that does not depend on τ is obtained by integrating $\widehat{\theta}_{\text{CGMM}}(\tau)$ against a measure $\pi(\tau)\,d\tau$ on Λ. Unfortunately, some parameters that could be identified from the full information set $\left\{h_t(\tau, \theta_0),\ \tau \in \mathbb{R}^d\right\}$ may no longer be identifiable from the reduced information set $\left\{h_{\tau,t}(u, \theta_0),\ u \in \mathbb{R}\right\}$. This leads us to consider a second approach that relies on a discrete subset of the full information set for the estimation of θ_0. Here, one draws a set of n indices $\tau_1, \ldots, \tau_n$ independently from a continuous distribution on $\mathbb{R}^d$, where n is large enough to ensure the identification of θ from $\{h_t(\tau_i, \theta_0),\ i = 1, \ldots, n\}$. A consistent estimator $\widehat{\theta}_{\text{GMM}}(\widetilde{\tau})$ is then obtained by minimizing a norm of $\frac{1}{T}\sum_t^T \widetilde{h}_t(\widetilde{\tau}, \theta_0)$, where $\widetilde{\tau} = (\tau_1, \ldots, \tau_n)$ and $\widetilde{h}_t(\widetilde{\tau}, \theta_0)$ is the n-vector of relevant moment conditions. A final estimator that does not depend on $\widetilde{\tau}$ is obtained by integrating $\widehat{\theta}_{\text{GMM}}(\widetilde{\tau})$ against a measure $\widetilde{\Pi}(\widetilde{\tau})\,d\widetilde{\tau}$ on $\left(\mathbb{R}^d\right)^n$. In either case, the final estimator consists of the aggregation of estimators obtained from samples of type $\left\{y_{\tau,t} = \tau' x_t\right\}_{t=1}^T$ generated from the frequency domain of the distribution of x_t and thus, it has the flavor of one obtained by a resampling method. It is also reminiscent of an indirect estimator because the underlying procedure converts a high dimensional model into a continuum of reduced information sets. As such, we refer to this estimator as the indirect CGMM (henceforth, ICGMM) estimator.

Three major issues are addressed regarding the ICGMM procedure. The first issue is related to the identification of θ_0 from the reduced information sets. The second issue involves the choice of the aggregating measure that warrants minimum variance for the ICGMM estimator. It appears that the optimal weighting scheme is closely related to the inverse of the covariance operators associated with the random element $\widehat{\theta}_{\text{CGMM}}(\tau)$ and $\widehat{\theta}_{\text{GMM}}(\widetilde{\tau})$. The third issue concerns the implementation of the efficient ICGMM. We propose an implementation strategy that relies on a combination of time domain and frequency domain resampling and we show that the feasible efficient ICGMM estimator converges in probability to its theoretical counterpart.

We perform two sets of Monte Carlo experiments that are aimed at comparing the performance of the ICGMM estimator to that of feasible benchmarks (e.g., Maximum Likelihood, CGMM or Smoothed Minimum Distance). The first set of experiments is based on a bivariate Gaussian IID model. We use this simple framework as a pretext to introduce a non-technical summary of the implementation steps of the ICGMM estimator. The second simulation study is based on an enriched version of a linear heteroscedastic model used in Cragg (1983). We summarize this model into a CMR that is turned into a continuum of unconditional moment restrictions for the purpose of applying the ICGMM procedure. The simulation results suggest that the ICGMM procedure compares favorably to the benchmarks and makes an efficient use of the information content of moment restrictions.

The remainder of the paper is organized as follows. Section 2 presents the general framework for implementing the ICGMM and introduces the necessary assumptions. Section 3 discusses the properties of standard CGMM estimators. Section 4 presents the derivation of the optimal aggregating weight for the ICGMM estimator. In Section 5, a feasible version of the efficient ICGMM estimator is presented and its asymptotic optimality established. For sake of clarity, we focus on IID models in Sections 2–5. The extension of the ICGMM to CMR, Markov and weakly dependent models is discussed in Section 6. Section 7 presents the Monte Carlo simulations and Section 8 concludes. The proofs of the propositions are gathered in the Appendix.

Please cite this article in press as: Kotchoni, R., The indirect continuous-GMM estimation. Computational Statistics and Data Analysis (2013), http://dx.doi.org/10.1016/j.csda.2013.09.023