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A likelihood ratio type test for invertibility in moving average processes

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A new test for invertibility of moving average processes is proposed. The test is based on an explicit local approximation of the likelihood ratio. A simulation study compares the power with two previously suggested tests: a score type test and a numerical likelihood ratio test. Local to the null of noninvertibility, the proposed test is seen to have better power properties than the score type test and its power is only slightly below that of the numerical likelihood ratio test. Moreover, the test is extended to an ARMA(p, 1) framework, by using it on the estimated residuals of a fitted AR(p) model. A simulation study for ARMA(1, 1) shows that when varying the AR parameter, the test has better size properties than the score type test.

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1. Introduction

We will study the moving average (MA) model

$$y_t = \varepsilon_t - \theta\varepsilon_{t-1}, \tag{1}$$

where $\theta \leq 1$, all ε_t are $NID\left(0, \sigma^2\right)$ for $t \geq 0$ and we have observations at $t = 1, \ldots, T$. Our focus is the test of noninvertibility, i.e. to test $H_0 : \theta = 1$ versus $H_1 : \theta < 1$. Traditionally, such tests are used to check for overdifferencing, see e.g. Saikkonen and Luukkonen (1993), Leybourne and McCabe (1994) and the discussion in Davis and Dunsmuir (1996).

Tanaka (1990) suggested a score type test, which he proved to be locally best invariant and unbiased (LBIU). He argued that Likelihood Ratio or Wald tests are not feasible since the maximum likelihood estimator (MLE) is not explicit in MA models, see e.g. Cryer and Ledolter (1981). In spite of this, Davis et al. (1995) and Davis and Dunsmuir (1996) exploited likelihood methods. They showed the interesting result that the limiting distribution of the MLE differs from that of the local maximizer (LM), which is the estimator obtained when maximizing the likelihood under the parametrization $\theta = 1 - \gamma/T$. Davis et al. (1995) also derived the asymptotic distribution of the Generalized Likelihood Ratio (GLR) test. Moreover, they compared finite sample and limiting powers of the score and GLR tests, as well as tests directly based on the LM and ML estimators. They found that GLR outperforms the other tests except for in the region $\gamma < 5$, where the score test is slightly better. Among the other two, the LM estimator based test uniformly outperforms the ML counterpart in the studied range of γ values. However, Tanaka (1990) presented a generalization of the score test to the general ARMA case, while Davis et al. (1995) did not provide such a generalization of the GLR test.

More recent work includes Davis and Song (2011), who extended the GLR test to the MA(2) case. Yabe (2012) generalized the asymptotic distribution results of Tanaka (1990), which were obtained for $\theta = 1 - \gamma/T$, to the moderate deviation case,

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where $\theta = 1 - \gamma/T^{\alpha}$ for some $\alpha \in (0, 1)$. Vougas (2008) proposed a new ML estimation method that avoids the pile-up phenomenon (the estimator equals one with positive probability). A test for invertibility is based on the suggested estimator.

In the present paper, we suggest an approximation of the Likelihood Ratio (LR) test which, as in LM estimation, is based on a local approximation of the parameter around 1. Simulations indicate that the power of our test is close to the power of the GLR test, and larger than the power of the other explicit one, the score test. We also generalize our test to invertibility testing in the ARMA(p, 1) case, by using it on the estimated AR residuals. A simulation study in the case $p = 1$ investigates the size performances of the three tests. For small T, the score type test is oversized for negative values of the AR parameter and undersized for positive values. For large T, the score type test works fairly well, although it may be severely oversized for values of the AR parameter close to −1. The numerical likelihood ratio test appears rather robust except for when the AR parameter is close to one, in which instance it may be severely oversized. Comparing to the other two, our test performs better than the score type test for negative AR parameter values and small sample size, and in other cases it is almost overall slightly better. So for a user who prefers a test statistic in explicit form, we recommend to use our test rather than the score test, since it has the best size and power properties among the two.

The rest of the paper is as follows. In Section 2, we derive the approximative LR test as well as some of its limiting properties. A simulation study is conducted in Section 3. Generalization to the ARMA(p, 1) model is discussed in Section 4. Under this framework, Section 5 contains a simulation comparison of small sample sizes of the studied tests, while Section 6 concludes.

2. An approximative LR test

Writing $\mathbf{y} = (y_1, \ldots, y_T)'$, $\boldsymbol{\varepsilon} = (\varepsilon_0, \varepsilon_1, \ldots, \varepsilon_T)'$, (1) is equivalent to

$$\mathbf{y} = \mathbf{H}'\boldsymbol{\varepsilon}, \tag{2}$$

where $\mathbf{H}$ is the $(T + 1) \times T$ matrix

$$\mathbf{H} \equiv \begin{pmatrix} -\theta & 0 & \cdots & 0 \\ 1 & -\theta & \ddots & \vdots \\ 0 & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & -\theta \\ 0 & \cdots & 0 & 1 \end{pmatrix}.$$

Hence, the log likelihood is

$$l\left(\theta, \sigma^2\right) = -\frac{T}{2}\log\left(2\pi\sigma^2\right) - \frac{\mathbf{y}'\boldsymbol{\Omega}^{-1}\mathbf{y}}{2\sigma^2} - \frac{1}{2}\log\left(\det \boldsymbol{\Omega}\right). \tag{3}$$

where $\boldsymbol{\Omega} = \mathbf{H}'\mathbf{H}$ is the covariance matrix of $\mathbf{y}$. Defining

$$\boldsymbol{\Omega}_1 = (\mathbf{I} - \theta\mathbf{L})\left(\mathbf{I} - \theta\mathbf{L}'\right), \tag{4}$$

where $\mathbf{I}$ and $\mathbf{L}$ are the identity and lag matrices of dimension T, we may write

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_1 + \theta^2\boldsymbol{\delta}_1\boldsymbol{\delta}_1', \tag{5}$$

where $\boldsymbol{\delta}_1 = (1, 0, \ldots, 0)'$. (In fact, if one would assume $\varepsilon_0 = 0$, then $\boldsymbol{\Omega}_1$ would be the covariance matrix of $\mathbf{y}$.) The MLE of σ^2, $\widehat{\sigma}^2$ say, fulfills

$$\widehat{\sigma}^2 = T^{-1}\mathbf{y}'\boldsymbol{\Omega}^{-1}\mathbf{y}. \tag{6}$$

Hence, (3) implies

$$l\left(\theta, \widehat{\sigma}^2\right) = -\frac{T}{2}\left\{\log(2\pi) + 1\right\} - \frac{T}{2}\log\left(\widehat{\sigma}^2\right) - \frac{1}{2}\log\left(\det \boldsymbol{\Omega}\right). \tag{7}$$

Moreover, we find from (5) that

$$\det \boldsymbol{\Omega} = \det \boldsymbol{\Omega}_1\left(1 + \theta^2\boldsymbol{\delta}_1'\boldsymbol{\Omega}_1^{-1}\boldsymbol{\delta}_1\right),$$

where $\det \boldsymbol{\Omega}_1 = 1$ and

$$\boldsymbol{\delta}_1'\boldsymbol{\Omega}_1^{-1}\boldsymbol{\delta}_1 = 1 + \theta^2 + \cdots + \theta^{2T-2} = \frac{1 - \theta^{2T}}{1 - \theta^2},$$

i.e.

$$\det \boldsymbol{\Omega} = 1 + \theta^2\frac{1 - \theta^{2T}}{1 - \theta^2} = \frac{1 - \theta^{2T+2}}{1 - \theta^2}.$$

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