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Variance clustering improved dynamic conditional correlation MGARCH estimators☆

Gian Piero Aielli, Massimiliano Caporin*

Department of Economics and Management Marco Fanno, University of Padova, Via del Santo 33, Padova, Italy

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ABSTRACT

It is well-known that the estimated GARCH dynamics exhibit common patterns. Starting from this fact the Dynamic Conditional Correlation (DCC) model is extended by allowing for a clustering structure of the univariate GARCH parameters. The model can be estimated in two steps, the first devoted to the clustering structure, and the second focusing on dynamic parameters. Differently from the traditional two-step DCC estimation, large system feasibility of the joint estimation of the whole set of model's dynamic parameters is achieved. A new approach to the clustering of GARCH processes is also introduced. Such an approach embeds the asymptotic properties of the univariate quasi-maximum-likelihood GARCH estimators into a Gaussian mixture clustering algorithm. Unlike other GARCH clustering techniques, the proposed method provides a natural estimator of the number of clusters.

1. Introduction

The Dynamic Conditional Correlation GARCH (DCC) model (Engle, 2002) has recently become one of the most popular tools for the estimation of multivariate asset volatility dynamics (for applications see, e.g., Cappiello et al. (2006), Billio et al. (2006), Billio and Caporin (2009), Franses and Hafner (2009) and Pesaran and Pesaran (2007); some theoretical results are in Engle and Sheppard (2001), McAleer et al. (2008) and Aielli (forthcoming)). The basic idea of the DCC modelling approach is to obtain first the variance process via simple univariate specifications, and, then to build a model for the correlation process from some appropriate function of the univariate variance standardized returns. As a direct advantage of such a modelling strategy, we obtain a two-step estimation procedure that is feasible with large systems: in the first step, the univariate variance processes are estimated one at a time, and, then, in the second step, the correlation process is estimated from the estimated standardized returns provided by the first step. Under appropriate specifications of the correlation process such as the cDCC specification adopted in this paper (Aielli, forthcoming), the two-step estimation procedure is shown to have desirable theoretical and empirical properties.

Most proposed extensions of the DCC model have been developed with the aim of providing more flexibility to the model of the correlation process. Two examples are the Asymmetric DCC model by Cappiello et al. (2006), and the Flexible DCC model by Billio et al. (2006); see also Billio and Caporin (2009), Engle and Kelly (2012) and Franses and Hafner (2009) for other applications. In particular, with the Flexible DCC model a rather rich parametrization of the correlation process is obtained by relying on *a priori* knowledge of the presence of some asset partition. The focus of this paper is different: we aim at improving the flexibility of the DCC modelling approach by focusing attention on the "variance side", rather than on the "correlation side", of the DCC model. Our final interest is to avoid, or reduce, the loss of efficiency (if any) that

☆ An appendix containing additional tables and figures can be found as a supplementary material of the electronic version of the paper.

* Corresponding author. Tel.: +39 049 8274258; fax: +39 049 8274211.
E-mail addresses: massimiliano.caporin@unipd.it, max.caporin@gmail.com (M. Caporin).

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potentially affects the traditional two-step DCC estimator; in fact, in that case, the variance processes are estimated one at a time and independently from the information embedded in the correlation dynamics. With this aim in mind, we take our cue from the well-known fact that the univariate estimates of GARCH dynamic parameters are often similar. We then allow the specification of the cDCC model to take advantage of this empirical finding by assuming that there can be assets in the selected asset sample that share common GARCH dynamics. We call this assumption *variance clustering* (VC) *assumption*, and the resulting model, VC-cDCC model. If the number of clusters is equal to the number of assets, we obtained the cDCC model as a special case of the VC-cDCC model. Given an estimate of the underlying asset variance clustering structure, the joint estimation of the whole set of model dynamic parameters – including the variance dynamic parameters and the correlation dynamic parameters – becomes possible.

Under a fully Bayesian framework, Bauwens and Rombouts (2007) recently addressed the estimation of the underlying GARCH dynamic clustering structure. The refined clustering approach proposed by the authors makes heavy use of Monte Carlo Markov Chain (MCMC) methods. As a result, their approach is computationally intensive, in particular with a large number of assets. A mixed frequentist-Bayesian approach is proposed by Brownlees (2009), in which the parameters of a set of GARCH processes are modelled as functions of observed regressors and unobserved idiosyncratic shocks. On the contrary, in our paper we adopt a fully frequentist approach, with the explicit aim of providing a fast and friendly clustering methodology, in accordance with the motivations behind the DCC estimation paradigm, as originally proposed by Engle (2002). Our clustering algorithm constitutes a new contribution to the literature on the clustering of GARCH processes, or, in more general terms, on the clustering of financial time series (see Liao (2005) for a survey on time series clustering). We know from Francq and Zakoïan (2010) and therein cited references that the asymptotic distribution of the univariate Quasi-maximum Likelihood (QML) GARCH estimator is Gaussian. Therefore, a possible choice is to consider a Gaussian mixture (GM) clustering framework. Unlike Bauwens and Rombouts (2007), who specify a GM model for the observed series as a consequence of the assumption of Gaussian conditionally distributed returns, our GM model is specific to the sample of the univariate QML estimates of the GARCH dynamic parameters. We thus change the focus of the GM clustering from the return process to the GARCH estimators. As a result, our approach is more flexible since we are working on a distribution-free GARCH framework where asymptotic properties of the QML univariate GARCH estimators are valid anyway. We also point out that, to estimate the peculiar mixture component covariance matrix implied by our GM model specification, we resort to an appropriate combination of standard univariate QML estimation outputs. Furthermore – and differently from other *ad hoc*, non-Bayesian GARCH clustering techniques (e.g., Otranto (2008, 2010)) – our clustering methodology logically leads to a BIC-based selection procedure for the identification of the number of clusters.

In order to deal with many assets, however, the reduction of the parameter dimensionality allowed by the variance clustering structure generally is not enough to allow the joint estimation of the variance/correlation dynamic parameters. The infeasibility is due to the fact that the dimensionality of the variance/correlation intercept parameters is $O(N^2)$, where N is the number of series. Following Aielli (forthcoming), we successfully solve this problem by resorting to an *ad hoc* generalized profile estimator (Severini, 1998). We thus treat the variance/correlation intercept parameters and the clustering structure as nuisance parameters, and the variance/correlation dynamic parameters as parameters of interest. We then replace the nuisance parameters in the model pseudo-likelihood with an estimator that can be easily computed conditionally on the parameters of interest. Thanks to such an estimation device, the joint estimation of the variance/correlation dynamic parameters becomes feasible (we call it the *joint* VC-cDCC estimator). The joint VC-cDCC estimator is shown to be superior to the traditional two-step cDCC estimator on the basis of both simulations and applications to real data.

For the joint VC-cDCC estimator to be feasible, it is required that the number of clusters is small, which, fortunately, is precisely what happens when dealing with large systems. An estimator of the VC-cDCC model that is feasible irrespective of the number of clusters is, however, also suggested in this paper (the *sequential* VC-cDCC estimator). With the sequential estimator, the asset variance dynamics are still estimated jointly, but separately from the correlation process. As objective functions for the variance dynamic parameters we adopt univariate composite likelihoods (Pakel et al., 2011). A special case of the sequential estimator is the traditional two-step cDCC estimator. The sequential estimator is extremely rapid to compute, and, surprisingly, does not cause any sensible loss of efficiency with respect to the joint estimator. From a theoretical perspective, this finding seems to suggest that the dynamic variance parameter and the dynamic correlation parameter are practically orthogonal (with respect to the adopted pseudo-likelihood).

The rest of the paper is organized as follows: Section 2 introduces the VC-cDCC model; Section 3 describes the VC-cDCC estimator while Section 4 contains the Monte Carlo simulations; Section 5 reports some applications to real data, and, finally, Section 6 concludes.

2. The VC-cDCC model

Before introducing the VC-cDCC model we review the DCC modelling approach. Denote by $y_t = [y_{1,t}, \ldots, y_{N,t}]'$ the $N \times 1$ vector of the asset returns at time t, and assume that

$$E_{t-1}[y_t] = 0, \qquad E_{t-1}[y_t y_t'] = H_t,$$

where $E_t[\cdot]$ is the conditional expectation on $y_t, y_{t-1}, \ldots$. The asset conditional covariance matrix can be written as

$$H_t = D_t^{1/2} R_t D_t^{1/2}, \tag{1}$$

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