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A joint test for structural stability and a unit root in autoregressions

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ABSTRACT

A test of the *joint* null hypothesis of parameter stability and a unit root within an ADF style autoregressive specification whose entire parameter structure is potentially subject to a structural break at an unknown time period is developed. The proposed test is a useful diagnostic tool for assessing the interactions of breaks and unit root type of nonstationarities in time series, in addition to offering a powerful device for detecting changes in persistence. As a byproduct the limiting behaviour of a related Wald statistic designed to test solely the null of parameter stability in an environment with a unit root is also obtained. These distributions are free of nuisance parameters and easily tabulated. The finite sample properties of the tests are assessed through a series of simulations, and an application to macroeconomic data illustrates their usefulness.

1. Introduction

A vast body of research in the recent time series econometrics literature has explored the interactions between nonlinear dynamics and unit root type of nonstationarities. Although initially nonlinearities and nonstationarities were often treated as separate and sometimes mutually exclusive phenomena, the development of new functional central limit theory amongst other technical tools has led to a growing body of research dealing with models in which both features could coexist. Under the structural break type of nonlinearities for instance, and starting with the early work of Perron (1989), there has been a vast literature on designing unit root tests that allowed for the presence of breaks in the underlying deterministic trend function of a series. One motivation for this line of research was the observation that the omission or misspecification of such trend breaks could lead to misleading inferences about the presence of unit roots. Important contributions in this area include Zivot and Andrews (1992) and Banerjee et al. (1992) and more recently Harris et al. (2009) and Kim and Perron (2009) amongst numerous others. The complications induced by the coexistence of structural breaks and unit roots have also triggered an interesting research agenda that instead focused on the impact of unit roots on Chow/Andrews type parameter stability tests and documented a spurious break phenomenon, whereby ignoring the presence of a unit root in an otherwise linear model was shown to frequently lead to the detection of spurious break points (see Bai (1998)). Despite the voluminous literature that explored these issues numerous open questions on the impact of nonstationarity on tests for structural breaks still remain.

In this paper our goal is to explore the joint interaction of structural change and unit roots by allowing the parameters of both the deterministic and stochastic components of an augmented Dickey–Fuller (ADF) type autoregressive model to be subject to a structural break. Unlike the existing literature that has mainly sought to robustify unit root inferences to trend breaks and other related features we instead concentrate on detecting the presence of parameter instability in an ADF specification whose autoregressive parameters may also be subject to structural breaks. More specifically, we are interested in exploring the properties of a Wald type test statistic designed to test the joint null hypothesis of parameter stability

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and a unit root within an ADF style autoregression. We view our test as a useful and practical diagnostic tool for further enhancing the existing apparatus on structural break and unit root testing. Subject to some confidence level for instance, a non-rejection of our joint null of a unit root and parameter stability may preclude the need to undertake further break point or related analyses. In addition, and unlike traditional unit root tests, our new test is also shown to have a strong ability to detect switches from a unit root to a stationary regime and vice versa. The relevance of such alternatives is extensively documented in the persistence change literature (see for instance Busetti and Taylor (2004), Kejriwal et al. (2011), Leybourne et al. (2003) and Leybourne and Taylor (2004), and references therein) as well as the work of Chong (2001), and as discussed below our proposed test displays a powerful ability to detect such occurrences. Besides the context of the unit root literature numerous related papers have also investigated tests of parameter stability in all the parameters of otherwise stationary regression models (Andrews, 1993; Bai, 1999, 2000; Bai and Perron, 1998). See also Dahlhaus (1997) for an alternative approach to modelling time variation in time series models.

As a byproduct of the above objectives we also derive the properties of a related Wald statistic whose sole purpose is to test the constancy of all the parameters characterising an ADF style autoregression when a unit root is imposed in the underlying model. This latter test statistic will help highlight the consequences of ignoring the presence of a unit root on commonly used tests for structural breaks, in addition to documenting the structural break based version of Theorem 1 in Caner and Hansen (2001).

Finally, we also view the motivation of this paper as following closely Caner and Hansen (2001), where the authors explored similar issues in models characterised by threshold effects as opposed to the structural break setting considered here. This comparison allows us to make interesting parallels between the two very different ways of capturing change. Note that it would be misleading to view our specification in (1) as a threshold model in which a time trend is used as the threshold variable, since these two types of nonlinearities generate very different asymptotics. This point is discussed extensively in Hansen (2000).

The plan of the paper is as follows. Section 2 presents our operating model and motivates the hypotheses of interest. Section 3 develops the large sample theory of our Wald type test statistics. Sections 4 and 5 provide numerical simulations and an application to macroeconomic data. Section 6 concludes. All proofs are relegated to the Appendix.

2. The model and hypotheses

Our operating model is given by the familiar ADF specification with all the parameters of its deterministic and stochastic components allowed to switch at some unknown time period k. Specifically, we consider

$$\Delta y_t = \begin{cases} \alpha_1 + \beta_1 t + \rho_1 y_{t-1} + \sum_{j=1}^{p} \gamma_{1j} \Delta y_{t-j} + e_t & t \le k \\ \alpha_2 + \beta_2 t + \rho_2 y_{t-1} + \sum_{j=1}^{p} \gamma_{2j} \Delta y_{t-j} + e_t & t > k, \end{cases} \tag{1}$$

with e_t denoting the random disturbance term. It is convenient to reformulate (1) in matrix form as

$$\Delta Y = X_1\theta_1 + X_2\theta_2 + e, \tag{2}$$

with $\Delta Y = (\Delta y_1, \ldots, \Delta y_T)'$. Letting $r_t = (1\ t)'$ and $z_{t-1} = (\Delta y_{t-1}, \ldots, \Delta y_{t-p})'$, X_1 above stacks the elements of $(r_t'\ y_{t-1}\ z_{t-1}')I(t \le k)$, X_2 those of $(r_t'\ y_{t-1}\ z_{t-1}')I(t > k)$ and $\theta_i = (\alpha_i\ \beta_i\ \rho_i\ \gamma_{i1}\ \cdots \gamma_{ip})'$ for $i = 1, 2$. Throughout this paper k will denote the unknown breakpoint location and for later use we also introduce the break fraction $\pi = \lim_{T\to\infty} k/T$ with $\pi \in [\underline{\pi}, \overline{\pi}] \subset (0, 1)$. For notational simplicity we will also refer to the two indicator functions as $I_1 \equiv I(t \le k)$ and $I_2 \equiv I(t > k)$. Note that given the context of our specification in (1) a more formal exposition would have involved a double array notation in the form $\{y_{T,t}\}$ for $t = 1, \ldots, T$. As it is the norm in this structural break literature we have dropped the dependence of our random variables on T, since it is of no consequence on our asymptotics and subsequent results. Letting $X = X_1 + X_2$ denote the matrix that stacks the elements $(r_t'\ y_{t-1}\ z_{t-1}')$ of the linear model it is also convenient to reparametrise (2) as

$$\Delta Y = X\theta_2 + X_1\Psi + e, \tag{3}$$

with $\Psi \equiv (\theta_1 - \theta_2)$.

Our main concern is to develop a test of the joint null of a unit root and the absence of a structural break in all the ADF parameters. We write this hypothesis as $H_0^A : \Psi = 0, \rho_1 = 0$. Under $p = 1$ in (1) for instance this notation is equivalent to writing $H_0^A : \alpha_1 = \alpha_2, \beta_1 = \beta_2, \gamma_{11} = \gamma_{21}, \rho_1 = \rho_2 = 0$. A non-rejection of this null would indicate support for the presence of a unit root in y_t together with the suitability of a linear autoregressive specification, while precluding the need to explore further the potential presence of breakpoints in some or all of the ADF parameters. In this sense, we view the implementation of a test such as $H_0^A : \Psi = 0, \rho_1 = 0$ as a useful diagnostic tool. Furthermore, as demonstrated below, we expect our test to display a strong ability to detect scenarios where ρ_i switches from zero to a stationary region such as $\{\rho_1 = 0, \rho_2 < 0\}$ or alternatively $\{\rho_1 < 0, \rho_2 = 0\}$.