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Regime switches in the dependence structure of multidimensional financial data

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ABSTRACT

Misperceptions about extreme dependencies between different financial assets have been an important element of the recent financial crisis, which is why regulating entities do now require financial institutions to account for different behavior under market stress. Such sudden switches in dependence structures are studied using Markov switching regular vine copulas. These copulas allow for asymmetric dependencies and tail dependencies in high dimensional data. Methods for fast maximum likelihood as well as Bayesian inference are developed. The algorithms are validated in simulations and applied to financial data. The results show that regime switches are present in the dependence structure and that regime switching models provide tools for the accurate description of inhomogeneity during times of crisis.

1. Introduction

In the reverberations of the recent financial crisis, regulators turned their attention towards the fact that financial time series exhibit different behavior under market stress. This has led to new requirements for financial institutions addressing this issue. For European financial institutions, the European Banking Authority (EBA) has introduced the Stressed Value at Risk (SVaR) in addition to the standard VaR measure (European Banking Authority, 2012). Here, the distributions of underlying risk factors which are used to calculate VaR have to be calibrated using a period of significant stress for the banks' portfolios to appropriately reflect different behavior of time series during these times.

In the literature on financial time series different behavior during times of market stress has long been recognized (e.g. in the seminal paper of Engle (1982) which shows that variances are not constant over time). A class of models addressing this are Markov switching (MS), also called regime switching or hidden Markov models (Hamilton, 1989). There, the behavior of a financial time series exhibits two or more distinct regimes, which correspond to different states of the economy. Which regime is present at a particular point of time is governed by an underlying hidden Markov process. These different states of the Markov model fit nicely into the regulatory framework requiring stressed and non-stressed market conditions to be incorporated in risk management. While there is a huge range of univariate or low dimensional applications of Markov switching models (Pelletier, 2006; Garcia and Tsafack, 2011; Chauvet and Hamilton, 2006; Cerra and Saxena, 2005; Hamilton, 2005), their full potential for the modeling of a multidimensional set of underlyings and indices as it is required in the risk management of financial institutions has not yet been explored. In particular, inference methods which assess estimation uncertainty must be developed. Further, characteristics of financial data such as asymmetric dependence and tail dependence (Longin and Solnik, 2001; Ang and Chen, 2002) must be incorporated appropriately. These are outside the world of the non tail-dependent Gaussian and symmetric Student's t distributions. A class of multivariate dependence models which can describe such asymmetries (Joe et al., 2010; Nikoloulopoulos et al., 2012) are regular vine (R-vine) copulas.

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They are based on a series of linked trees called *R*-vine and have been introduced by Bedford and Cooke (2001, 2002) extending ideas of Joe (1996). *R*-vine distributions are built up hierarchically from bivariate copulas as building blocks and they have proven to be a tractable model in the multidimensional setting since also inference can be performed exploiting that hierarchical structure (Aas et al., 2009).

Our contribution is to introduce a general Markov switching *R*-vine (MS–RV) copula model and to develop efficient inference techniques. In particular, we go beyond the MS copula model of Chollete et al. (2009) in that we use the full flexibility of *R*-vine models. This superior flexibility will allow us to use truncation techniques (see Brechmann et al. (2012) and references therein), leading to a parsimonious parametrization of the model. Secondly, we develop an approximative Expectation–Maximization (EM) type procedure in the Maximum Likelihood (ML) framework which allows for fast parameter estimation and is scalable to high dimensional applications. The algorithm is based on the sequential estimation procedure developed by Aas et al. (2009), which has been shown to be asymptotically consistent by Hobæk Haff (2013). To address the issue of quantifying uncertainty, we further consider parameter inference for a prespecified MS–RV model in a Bayesian setup. For this, the algorithm of Min and Czado (2010), who consider Bayesian inference for a structural subclass of *R*-vine copulas, is generalized and extended to incorporate inference about the underlying Markov structure. In particular, we can also compute credible intervals for the probability of being in a given regime at a given point of time. While most existing models for time-varying dependence do not allow to quantify the uncertainty in the time variability of parameters, our Bayesian estimation procedure enables us to do so.

In order to demonstrate the applicability and performance of our procedures for parameter estimation, we perform a simulation study and investigate an application to exchange rate data. We also show how model selection can be performed for time-varying dependence structures by conducting a rolling window analysis and compare different models using the Bayesian deviance information criterion (DIC, Spiegelhalter et al., 2002). The remainder is structured as follows: Section 2 introduces the MS–RV model by first introducing *R*-vine distributions in Section 2.1 and then combining them with an underlying Markov structure in Section 2.2. The focus of Section 3 is parameter estimation, the stepwise procedure is discussed in Section 3.1 and Bayesian estimation in Section 3.2. In Section 4, we present the simulation results before turning to the empirical application in Section 5. Section 6 gives an outlook to directions of future research.

2. The Markov switching regular vine copula model

2.1. Regular vine distributions

R-vines as a graph theoretic tool for the construction of multivariate distributions have been introduced by Bedford and Cooke (2001, 2002). An *R*-vine $\mathcal{V}$ on d variables, which consists of a sequence of connected trees $T_1, \ldots, T_{d-1}$, with nodes N_i and edges E_i, $1 \leq i \leq d-1$, satisfies the following properties (Bedford and Cooke, 2001):

1. T_1 is a tree with nodes $N_1 = \{1, \ldots, d\}$ and edges E_1.
2. For $i \geq 2$, T_i is a tree with nodes $N_i = E_{i-1}$ and edges E_i.
3. If two nodes in T_{i+1} are joined by an edge, the corresponding edges in T_i must share a common node. (*proximity condition*)

There are two popular subclasses of *R*-vines. We call an *R*-vine *Canonical vine* (*C*-vine) if in each tree T_i there is one node which has edges with all $d-i$ other nodes. It is called *Drawable vine* (*D*-vine) if each node has at most two edges. Examples of regular vine tree structures can be found for example in Czado (2010) or in the Appendix. The notation we employ throughout our paper follows Czado (2010). Let $\mathbf{X} = (X_1, \ldots, X_d)$ be a random vector with marginal densities $f_1, \ldots, f_d$, respectively. To build up a statistical model using the *R*-vine, we associate to each edge $j(e), k(e)|D(e)$ in E_i, for $1 \leq i \leq d-1$, a bivariate copula density $c_{j(e),k(e)|D(e)}$. We call $j(e)$ and $k(e)$ the *conditioned set* while $D(e)$ is the *conditioning set*. Let $\mathbf{X}_{D(e)}$ denote the subvector of $\mathbf{X}$ determined by the set of indices $D(e)$. For the definition of the R-*vine distribution* we associate the bivariate copula densities $c_{j(e),k(e)|D(e)}$ with the conditional densities of $X_{j(e)}$ and $X_{k(e)}$ given $\mathbf{X}_{D(e)}$ equal to $c_{j(e),k(e)|D(e)}(F_{j(e)|D(e)}(x_{j(e)}|\mathbf{x}_{D(e)}), F_{k(e)|D(e)}(x_{k(e)}|\mathbf{x}_{D(e)}))f_{j(e)|D(e)}f_{k(e)|D(e)}$. In general, $c_{j(e),k(e)|D(e)}$ can depend on the values of variables which are conditioned on. To keep the number of parameters tractable, it is usually assumed that the conditional copula is constant, i.e. $c_{j(e),k(e)|D(e)}(.,.|\mathbf{x}_{D(e)}) = c_{j(e),k(e)|D(e)}(.,.)$, as discussed in Stöber et al. (2012), Haff et al. (2010) and Acar et al. (2012). The joint density of $\mathbf{X}$ is then uniquely determined by

$$f_{1,\ldots,d}(x_1, \ldots, x_d) = \prod_{i=1}^{d} f_i(x_i) \cdot \prod_{i=1}^{d-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)}(F_{j(e)|D(e)}(x_{j(e)}|\mathbf{x}_{D(e)}), F_{k(e)|D(e)}(x_{k(e)}|\mathbf{x}_{D(e)})) \quad (1)$$

as shown by Bedford and Cooke (2001). If the marginal densities are uniform on [0, 1], we call the distribution in (1) an *R*-vine copula. Given an *R*-vine $\mathcal{V}$, a set of corresponding parametric bivariate copulas $\mathbf{B}$ and their parameter vector $\boldsymbol{\theta}$, we denote the *R*-vine copula density by $c(.|\mathcal{V}, \mathbf{B}, \boldsymbol{\theta})$.

While also other iterative decompositions of a multivariate density into bivariate copulas and marginal densities are possible, *R*-vine distributions have the particularly appealing feature that the values for $F(x_{j(e)}|\mathbf{x}_{D(e)})$ and $F(x_{k(e)}|\mathbf{x}_{D(e)})$ appearing in Eq. (1) can be derived recursively without high dimensional integrations (see Dißmann et al., 2013 for details).

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