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A fluctuation test for constant Spearman's rho with nuisance-free limit distribution

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ABSTRACT

A CUSUM type test for constant correlation that goes beyond a previously suggested correlation constancy test by considering Spearman's rho in arbitrary dimensions is proposed. Since the new test does not require the existence of any moments, the applicability on usually heavy-tailed financial data is greatly improved. The asymptotic null distribution is calculated using an invariance principle for the sequential empirical copula process. The limit distribution is free of nuisance parameters and critical values can be obtained without bootstrap techniques. A local power result and an analysis of the behavior of the test in small samples are provided.

1. Introduction

Recently, Wied et al. (2012) proposed a fluctuation test for constant correlation based on the Bravais–Pearson correlation coefficient. The test, which will be referred to as BPC test in the following, is for example useful in financial econometrics to examine changes in the correlation between asset returns over time. Longin and Solnik (1995) and Krishan et al. (2009) discuss the relevance of this question. The test complements former approaches by e.g. Galeano and Peña (2007) and Aue et al. (2009). However, one major drawback of this test is the fact that the limit distribution is derived under the condition of finite fourth moments (similar to Aue et al., 2009). This is a critical assumption because the existence of fourth moments in usually heavy-tailed financial returns is doubtful; see e.g. Grabchak and Samorodnitsky (2010), Krämer (2002) and Amaral et al. (2000).

This paper presents a fluctuation test for constant correlation based on Spearman's rho which imposes no conditions on the existence of moments.

There are several advantages of Spearman's rho compared to the Bravais–Pearson correlation: in many situations, e.g. if the data is non-elliptical, the Bravais–Pearson correlation may not be an appropriate measure for dependence. It is confined to measuring linear dependence, while the rank-based dependence measure Spearman's rho quantifies monotone dependence. If the second moments do not exist, the Bravais–Pearson correlation is not even defined, while Spearman's rho does not require any moments.

Spearman's rho is probably the most common rank-based dependence measure in economic and social sciences; see e.g. Gaißler and Schmid (2010), who propose tests for equality of rank correlations, and the references herein. In addition,

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Spearman's rho often performs better in terms of robustness than the Bravais–Pearson correlation. Embrechts et al. (2002) discuss several other pitfalls and possible problems for a risk manager who simply applies the Bravais–Pearson correlation.

Therefore, it is natural in the context of testing for changes in the dependence structure of random vectors to extend the BPC test to a test for constant Spearman's rho. As expected from the theory of dependence measures, this test is applicable in more situations: it has a much better behavior in the presence of outliers and there are no conditions on the existence of moments. In addition, the test is applicable in arbitrary dimensions, while the BPC test is designed for bivariate random vectors. Similarly to the BPC test, the test bases on successively calculated empirical correlation coefficients in the style of Ploberger et al. (1989), Lee et al. (2003) or Galeano (2007).

The limit distribution of our test statistic is the supremum of the absolute value of a Brownian bridge. This immediately provides critical values without any bootstrap techniques. We impose a strong mixing assumption for the dependence structure. The proof relies on an invariance principle for multivariate sequential empirical processes from Bücher and Volgushev (forthcoming).

By using the copula-based expression for Spearman's rho from Schmid and Schmidt (2007) or Nelsen (2006), we get quite another contribution with our test, i.e. an extension of the copula constancy tests proposed by Busetti and Harvey (2011) and Krämer and van Kampen (2011). Since copula models are frequently used in financial econometrics (see e.g. Manner and Reznikova, 2011 and Giacomini et al., 2009), such tests for structural change are important in this area. However, they are restricted to the case of testing for copula constancy in one particular quantile, e.g. the 0.95-quantile. This might be an important null hypothesis as well, but our test now (indirectly) allows for testing constancy of the whole copula by integrating over it. We therefore reject the null hypothesis of constant Spearman's rho (which is closely connected to the null hypothesis of an overall constant copula) if the integral over it fluctuates too much over time. The problem of testing constancy of the whole copula has recently been dealt with in the literature (see Kojadinovic and Rohmer, 2012, Bücher and Ruppert, 2013, van Kampen and Wied, 2012 and Rémillard, 2010). All these approaches, however, need the computationally intensive bootstrap for approximating the limit distribution. With our approach, we need much less computational time for calculating the critical values.

The paper is organized as follows: Section 2 presents our test statistic and the asymptotic null distribution, Section 3 considers local power, Section 4 presents Monte Carlo evidence about the behavior of the test in small samples. Section 5 compares our new test with the BPC test in terms of robustness by a simulation study and an empirical application. Finally, Section 6 concludes. All proofs are in the Appendix.

2. Test statistic and its asymptotic null distribution

In this section, we present the test statistic and the limit distribution of our test under the null. First, we introduce some notation: $(\mathbf{X}_1, \ldots, \mathbf{X}_n)$ are d-dimensional random vectors on the probability space $(\Omega, \mathfrak{A}, \mathsf{P})$ with $\mathbf{X}_j = (X_{1,j}, \ldots, X_{d,j}), j = 1, \ldots, n$. Regarding the dependence structure, we impose the following assumption:

(A1) $\mathbf{X}_1, \ldots, \mathbf{X}_n$ are α-mixing with mixing coefficients α_j satisfying

$$\sum_{j=1}^{\infty} j^2 \alpha_j^{\gamma/(4+\gamma)} < \infty$$

for some $\gamma \in (0, 2)$.

This dependence assumption is similar to the assumption made in Inoue (2001) and holds in most econometric models relevant in practice, e.g. for ARMA- and GARCH-processes under mild additional conditions; see e.g. Carrasco and Chen (2002).

The vectors $\mathbf{X}_j, j = 1, \ldots, n$, have joint distribution functions F^j with

$$F^j(\mathbf{x}) = \mathsf{P}(X_{1,j} \le x_1, \ldots, X_{d,j} \le x_d), \quad \mathbf{x} = (x_1, \ldots, x_d) \in \mathbb{R}^d,$$

and marginal distribution functions $F_{i,j}(x) = \mathsf{P}(X_{i,j} \le x)$ for $x \in \mathbb{R}$ and $i = 1, \ldots, d$ which are assumed to be continuous.

According to Sklar's (1959) theorem, there exists a unique copula function $C_j : [0, 1]^d \to [0, 1]$ of $\mathbf{X}_j$ with

$$F^j(\mathbf{x}) = C_j(F_{1,j}(x_1), \ldots, F_{d,j}(x_d))$$

and

$$C_j(\mathbf{u}) = F_j(F_{1,j}^{-1}(u_1), \ldots, F_{d,j}^{-1}(u_d)), \quad \mathbf{u} = (u_1, \ldots, u_d) \in [0, 1]^d,$$

where F^{-1} is the generalized inverse function; see e.g. Schmid and Schmidt (2007).

In terms of the copula, Spearman's rho is defined as

$$\rho_j = h(d) \cdot \left(2^d \int_{[0,1]^d} C_j(\mathbf{u}) d\mathbf{u} - 1 \right)$$

with $h(d) = \frac{d+1}{2^d-(d+1)}$; see Schmid and Schmidt (2007) or Nelsen (2006). There are also other possibilities to define Spearman's rho in higher dimensions (see Schmid and Schmidt, 2007 and Quessy, 2009), but we focus on this expression