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Bayesian analysis of a Gibbs hard-core point pattern model with varying repulsion range

T. Rajala [a,*], A. Penttinen [b]

[a] *Helsinki Institute for Information Technology, Department of Information and Computer Science, P.O. Box 15400, 00076 Aalto, Finland*

[b] *Department of Mathematics and Statistics, P.O. Box 35, 40014 University of Jyväskylä, Finland*

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ABSTRACT

A Bayesian solution is suggested for the modelling of spatial point patterns with inhomogeneous hard-core radius using Gaussian processes in the regularization. The key observation is that a straightforward use of the finite Gibbs hard-core process likelihood together with a log-Gaussian random field prior does not work without penalisation towards high local packing density. Instead, a nearest neighbour Gibbs process likelihood is used. This approach to hard-core inhomogeneity is an alternative to the transformation inhomogeneous hard-core modelling. The computations are based on recent Markovian approximation results for Gaussian fields. As an application, data on the nest locations of Sand Martin (*Riparia riparia*) colony[1] on a vertical sand bank are analysed.

1. Introduction

Observed spatial point patterns are usually assumed to be spatially stationary or, in the finite case, spatially homogeneous. However, data consisting of point locations may reveal inhomogeneity in point density, in local structure, or in both.

The present paper deals with Bayesian modelling of finite spatial point patterns with inhibition between the points. It is assumed that the range of inhibition is spatially inhomogeneous, and this inhomogeneity is modelled through a latent Gaussian process. As a result a posterior description of the background inhomogeneity is obtained as well as predictions for further points in the point pattern.

A motivating example consists of a point pattern of Sand Martin's (*Riparia riparia*) nests on a vertical sand bank, see Fig. 1. The figure reveals that the nest holes are highly packed but the packing density varies, partly according to the unobserved variation of sand composition in the bank. Further examples can be found e.g. in the fields of solid matter physics (Hahn et al., 2003) and physiology (Nielsen and Vedel Jensen, 2004).

The mainstream of the modelling of inhomogeneous point patterns is focused on the intensity estimation for inhomogeneous Poisson processes, see e.g. the discussion in Guttorp and Thorarinsdottir (2011). Because of the nature of the Poisson process, local structures such as inhibition between the points are excluded. Illian et al. (in press) show that interaction-like effect can be introduced to the Poisson process by adding spatially continuous morphological summaries as "constructed covariates" for the intensity. In the non-Poisson setting, Vedel Jensen and Nielsen (2000) consider a model where a homogeneous Markov point process is transformed to an inhomogeneous one. This process class is called

* Correspondence to: Department of Information and Computer Science, P.O. Box 15400, 00076 Aalto, Finland. Tel.: +358 400 695 103.
E-mail addresses: tuomas.rajala@iki.fi (T. Rajala), antti.k.penttinen@jyu.fi (A. Penttinen).

1 Dataset is attached to the online version.

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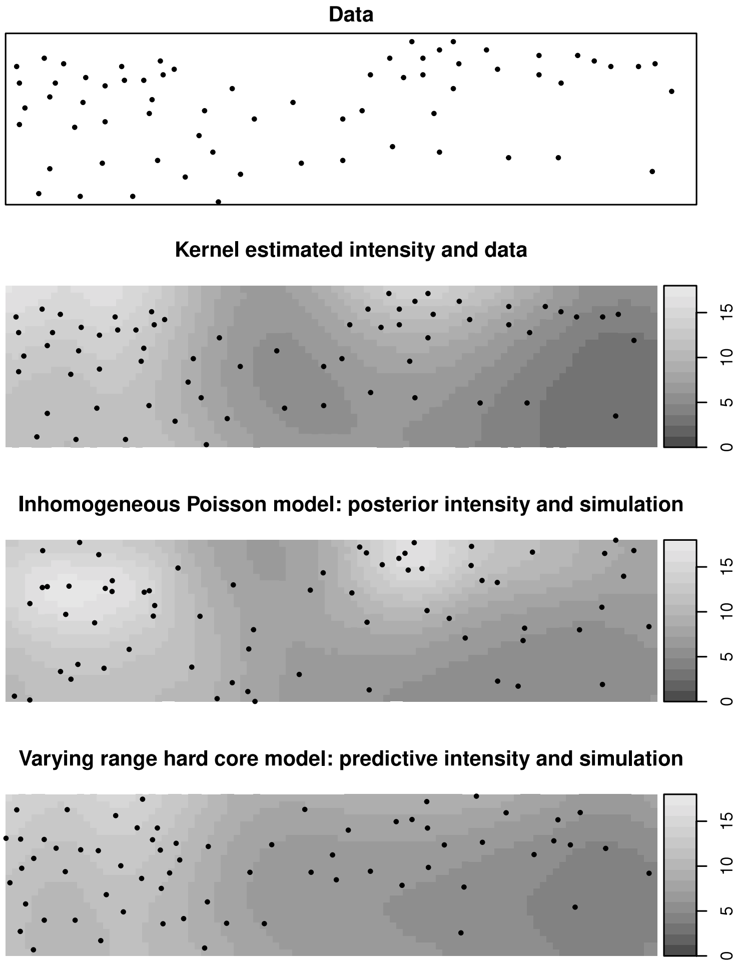

Fig. 1. From the top down: Locations of Sand Martin's nests on a sand bank as will be described in Section 4; data overlaid on a kernel smoothed intensity field; a simulation of an inhomogeneous Poisson process model overlaid on the mean posterior intensity field; simulation from the new varying range hard-core model overlaid on a mean posterior predictive intensity field.

"Transformation inhomogeneous Markov" (TIM), see also Nielsen and Vedel Jensen (2004) and Hahn et al. (2003). The use of transformation is a modification of the idea by Sampson and Guttorp (1992) for geostatistical random fields, preceded by the time acceleration models in survival analysis. The inference for the TIM model is based on finding the most likely transformation. As an alternative to TIM modelling, Berthelsen and Møller (2008) apply Markov point processes defined in terms of inhomogeneous self-potential and pairwise interaction function giving a Bayesian solution.

In the present paper it is assumed that the points in a point pattern have a hard-core with smoothly varying radius. The pattern can be thought to be the set of centre points of non-intersecting spherical objects of variable size. This model is called a *hard spheres* model, and the radius at a location is tied to the physical size of the object, see e.g. Månsson and Rudemo (2002). An alternative construction is to associate each point with a radius and define the hard-core area by excluding other points within the sphere. Then the variable radius can be addressed to the point location. This process is an extension of the Markov *hard-core* process (finite Gibbs hard-core process). We adopt the latter point of view.

The Markov hard-core process is defined by the self-potential and pairwise interaction, and both of these contribute to the Papangelou conditional intensity function, see Illian et al. (2008, p. 149) and van Lieshout (2000, p. 39). However, if we are close to the case of high packing, then the pairwise interaction (hard-core) dominates, and the spatial distribution of the hard-core radius has a primary effect. Our suggestion for modelling varying radius hard-core Gibbs processes is by means of latent smooth Gaussian processes, see Rasmussen and Williams (2006): the hard-core distance for each point is determined by a random function depending on this Gaussian process.

Two problems will be met in fitting the varying radius hard-core process. The first problem is the intractable scaling factor in the hard-core Markov point process likelihood which in this case depends also on the unknown hard-core radius function. The known solutions, importance sampling applied by Bognar (2005) and auxiliary variable method by Møller et al. (2006), lead to demanding computation. Also, the approximation of the likelihood with the pseudo-likelihood function (Jensen and Møller, 1991) depends on the unknown latent function. Secondly, although the synthesis (i.e. simulation of the