Contents lists available at ScienceDirect
Discrete Applied Mathematics
journal homepage: www.elsevier.com/locate/dam

The 1-good-neighbor connectivity and diagnosability of Cayley graphs generated by complete graphs☆

Mujiangshan Wang[a,*], Yuqing Lin[a], Shiying Wang[b]

[a] School of Electrical Engineering & Computer Science, The University of Newcastle, NSW 2308, Australia
[b] Henan Engineering Laboratory for Big Data Statistical Analysis and Optimal Control, School of Mathematics and Information Science, Henan Normal University, Xinxiang, Henan 453007, PR China

ARTICLE INFO

Article history:
Received 30 November 2015
Received in revised form 4 February 2017
Accepted 23 February 2017
Available online xxxx

Keywords:
Interconnection network
Graph
Diagnosability
PMC model
MM* model
Cayley graph
1-Good-neighbor connectivity
1-Good-neighbor diagnosability

ABSTRACT

Diagnosability is a significant metric to measure the reliability of multiprocessor systems. In 2012, a new measure for fault tolerance of the system was proposed by Peng et al. This measure is called the g-good-neighbor diagnosability that restrains every fault-free node to contain at least g fault-free neighbors. The Cayley graph CK_n generated by the complete graph K_n has many good properties as other Cayley graphs. In this paper, we show that the connectivity of CK_n is $\frac{n(n-1)}{2}$, the 1-good-neighbor connectivity of CK_n is n^2-n-2 and the 1-good-neighbor diagnosability of CK_n under the PMC model is n^2-n-1 for $n \geq 4$ and under the MM* model is n^2-n-1 for $n \geq 5$.

1. Introduction

In many multiprocessor systems, we usually use interconnection networks (networks for short) as underlying topologies of these multiprocessor systems and a network is usually represented by a graph whose nodes represent processors and links represent communication links between processors. We use graphs and networks interchangeably. It is of obvious significance to study the topological properties of the network corresponding to a multiprocessor system. Since there is a possibility that processors (nodes) fail and create faults in the system, the node fault identification for such systems must be considered. Firstly we should identify the faulty processors from the fault-free ones in order to deal with faults. The identification process is called the diagnosis of the system. A system is said to be t-diagnosable if all faulty processors can be identified without replacement, provided that the number of presented faults does not exceed t. The diagnosability $t(G)$ of a system G is the maximum value of t such that G is t-diagnosable [6,7,11]. For a t-diagnosable system, Dahbura and Masson [6] proposed an algorithm with time complex $O(n^{2.5})$, which can effectively identify the set of faulty processors.

Several diagnosis models were proposed to identify the faulty processors. One major approach is the PMC diagnosis model introduced by Preparata et al. [15]. Using mutual test between two linked processors can achieve the diagnosis of the system. Another major approach, namely the comparison diagnosis model (MM model), was proposed by Maeng and

☆ This work is supported by the National Science Foundation of China (61370001).
* Corresponding author.
E-mail addresses: wmjsdr@126.com (M. Wang), wangshiying@htu.cn (S. Wang).

http://dx.doi.org/10.1016/j.dam.2017.02.023

Malek [13]. In the MM model, to diagnose the system, a node sends the same task to two of its neighbors, and then compares their responses. In 2005, Lai et al. [11] introduced a restricted diagnosability of the system called conditional diagnosability. They considered the situation that no faulty set can contain all the neighbors of any vertex in the system. In 2012, Peng et al. [14] proposed a new measure for fault diagnosis of the system, namely, the g-good-neighbor diagnosability (which is also called the g-good-neighbor conditional diagnosability), which requires that every fault-free node has at least g fault-free neighbors. In [14], they studied the g-good-neighbor diagnosability of the n-dimensional hypercube under the PMC model. In [20], Wang and Han studied the g-good-neighbor diagnosability of the n-dimensional hypercube under the MM* model. Yuan et al. [24,25] studied that the g-good-neighbor diagnosability of the k-ary n-cube ($k \geq 3$) under the PMC model and MM* model. The Cayley graph $C\Gamma_n$ generated by the transposition graph Γ_n has recently received considerable attention. In [19,21], Wang et al. studied the g-good-neighbor diagnosability of $C\Gamma_n$ under the PMC model and MM* model for $g = 1, 2$. In this paper, it is proved that (a) the connectivity of the Cayley graph CK_n generated by the complete graph K_n is $\frac{n(n-1)}{2}$; (b) the 1-good-neighbor connectivity of CK_n is $n^2 - n - 2$; (c) the 1-good-neighbor diagnosability of CK_n under the PMC model is $n^2 - n - 1$, where $n \geq 4$; (d) the 1-good-neighbor diagnosability of CK_n under the MM* model is $n^2 - n - 1$, where $n \geq 5$.

2. Preliminaries

In this section, we discuss some definitions and notations such as the Cayley graph generated by the complete graph, the PMC model and MM* model.

2.1. Notations

A simple graph $G = (V, E)$ could be used to model a multiprocessor system, whose processors are represented as V and communication links are represented as E. Given a subset V' of V such that $|V'| > 0$, we denote the subgraph induced by V' in G by $G[V']$. For $G[V']$, $V(G[V']) = V'$ and $E(G[V'])$ is the set of all the edges of G with both endpoints in V'. The degree $d_G(v)$ of a vertex v is the number of edges incident with v. The minimum degree of a vertex in G is denoted by $\delta(G)$. For each vertex v, the neighborhood $N_G(v)$ of v in G is defined to be the set of vertices adjacent to v. Let $S \subseteq V$. $N_G(S)$ denotes the set $\bigcup_{v \in S} N_G(v) \setminus S$. For neighborhoods and degrees, it is usual to omit the subscript for the graph when no confusion arises. If a graph G is k-regular, then for each of its vertex v, we have $d_G(v) = k$. Let G be a connected graph. The connectivity $\kappa(G)$ of G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left when G is complete. For a connected graph $G = (V, E)$, we call a faulty set $F \subseteq V$ a g-good-neighbor faulty set if $|N(v) \cap (V \setminus F)| \geq g$ for every vertex v in $V \setminus F$. A g-good-neighbor cut of a graph G is a g-good-neighbor faulty set F such that $G - F$ is disconnected. The minimum cardinality of g-good-neighbor cuts is defined as the g-good-neighbor connectivity of G, denoted by $\kappa^{(g)}(G)$. A connected graph G is said to be g-good-neighbor connected if G has a g-good-neighbor cut. The terminology and notation not defined here are referred to [2].

2.2. The PMC model and MM* model

Under the PMC model [15], to diagnose a system $G = (V, E)$, two adjacent nodes in G are able to perform mutual tests. For two adjacent nodes u and v in V, we use the ordered pair (u, v) to represent the test performed by u on v. The outcome of a test (u, v) is 1 (resp. 0) if v is evaluated by u as faulty (resp. fault-free). In the PMC model, the testing result is usually assumed as reliable (resp. unreliable) if the node u is fault-free (resp. faulty). A test assignment T for a system G is a collection of tests for every adjacent pair of vertices. It can be modeled as a directed testing graph $(V(G), L)$, where $(u, v) \in L$ implies that u and v are adjacent in G. The collection of all test results for a test assignment T is called a syndrome. Formally, a syndrome is a function $\sigma : L \mapsto \{0, 1\}$.

The set of all faulty processors in the system is called a faulty set. This can be any subset of $V(G)$. For a given syndrome σ, a subset of vertices $F \subseteq V(G)$ is said to be consistent with σ if syndrome σ can be produced from the situation that, for any $(u, v) \in L$ such that $u \in V \setminus F$, $\sigma(u, v) = 1$ if and only if $v \in F$. This means that F is a possible set of faulty processors. Since a test outcome produced by a faulty processor is unreliable, a given set F of faulty vertices may produce a lot of different syndromes. On the other hand, different faulty sets may produce the same syndrome. Let $\sigma(F)$ denote the set of all syndromes which F is consistent with.

Under the PMC model, two distinct sets F_1 and F_2 in $V(G)$ are said to be indistinguishable if $\sigma(F_1) \cap \sigma(F_2) \neq \emptyset$, otherwise, F_1 and F_2 are said to be distinguishable. Besides, we say (F_1, F_2) is an indistinguishable pair if $\sigma(F_1) \cap \sigma(F_2) \neq \emptyset$; else, (F_1, F_2) is a distinguishable pair.

For the MM model [13,16], the diagnosis is carried out by sending the same testing task from one processor to a pair of processors and comparing their responses. Under the MM model, the following assumptions are made: (a) all faults are permanent; (b) a faulty processor produces incorrect outputs for each of its given tasks; (c) the output of a comparison performed by a faulty processor is unreliable. The comparison scheme of a system $G = (V, E)$ is modeled as a multigraph, denoted by $M(V(G), L)$, where L is the labeled-edge set. A labeled edge $(u, v)_w \in L$ represents a comparison in which two vertices u and v are compared by a vertex w, which implies $uw, vw \in E(G)$. The collection of all comparison