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General cut-generating procedures for the stable set polytope

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ABSTRACT

We propose general separation procedures for generating cuts for the stable set polytope, inspired by a procedure by Rossi and Smriglio and applying a lifting method by Xavier and Campêlo. In contrast to existing cut-generating procedures, ours generate both rank and non-rank valid inequalities, hence they are of a more general nature than existing methods. This is accomplished by iteratively solving a lifting problem, which consists of a maximum weighted stable set problem on a smaller graph. Computational experience on DIMACS benchmark instances shows that the proposed approach may be a useful tool for generating cuts for the stable set polytope.

1. Introduction

Let $G = (V, E)$ be an undirected graph with node set V and edge set E. A *stable set* in G is a subset of pairwise non-adjacent vertices of G. Given a graph G, the *maximum stable set (MSS)* problem asks for a stable set S in G of maximum cardinality. The *stability number of* G is this maximum cardinality and is denoted by $\alpha(G)$. The MSS problem is computationally hard to solve in practice, being in NP-Hard, and is also hard to approximate, unless the graph G has some special structure. For an arbitrary input graph G, a number of exact methods have been developed to solve it through several combinatorial or mathematical programming-based techniques. For a survey of these theoretical and practical aspects of the MSS problem, see [3] and references therein. See also [27] for strong non-approximability results on the problem.

Enumerative combinatorial algorithms have shown to be efficient to solve the MSS problem exactly for moderately sized graphs (for an overview, see [25]). Typically, such algorithms perform a search in a tree with the employment of simple and fast, but still effective, bounding procedures for pruning purposes. In this vein, the most successful approach involves the use of approximate colorings of selected subgraphs of $\bar{G}$ (the complement of G). This bound is based on the following remark: if $\bar{G}$ admits an ℓ-coloring, then $\alpha(G) \leq \ell$. This is a relatively weak bound and, consequently, the procedure to compute it is generally applied at numerous nodes of the search tree. However, it can be computed quickly by means of a greedy coloring heuristic implemented with bit parallelism operations [7,19,22].

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An alternative to combinatorial algorithms is the use of sophisticated mathematical programming techniques to handle the combinatorial properties of the polytope associated with the formulation $\alpha(G) = \max\{\sum_{v\in V} x_v \mid x_u + x_v \leq 1, uv \in E, x_v \in \{0,1\}, v \in V\}$. Although combinatorial methods for the MSS problem from the literature outperform mathematical programming-based algorithms devised so far (at least for dense and medium dense graphs, see [10] for such comparisons), it is of great interest to continue the search for efficient polyhedral methods for this problem. Despite its natural theoretical relevance, there are other motivations of algorithmic nature, namely: (a) the algorithms can be easily extended to the weighted version of the MSS problem, (b) MSS constraints frequently appear as a sub-structure in many combinatorial optimization problems, (c) in many situations, probing strategies give MSS valid inequalities on conflicting variables for general mixed integer programs (see, e.g., [1,4]), and (d) real applications may need specific versions of the MSS problem with additional constraints. In this context, procedures for valid inequalities generation often turn out to be effective.

There are two main directions of research when polyhedral techniques, in particular procedures for valid inequalities generation, are concerned. The first direction is usually referred to as the *lift-and-project* method [6], which consists of three steps: first, a lifting operator is applied to the initial formulation to obtain a lifted formulation in a higher dimensional space; second, the lifted formulation is strengthened by means of additional valid inequalities; and third, a strengthened relaxation of the initial formulation is finally obtained as a result of an appropriate projection of the strengthened lifted formulation onto the original space. Several upper bounds for $\alpha(G)$ have been stated in connection of this method, such as the ones based on semidefinite programming (SDP, for short) relaxations described in [8,13], which can be rather time-consuming to compute in practice. More recently, a new relaxation was introduced in [9] which preserves some theoretical properties of SDP relaxations in generating effective cuts but is computationally more tractable for a range of synthetic instances.

The second direction of research consists of searching for cuts within the original model variables, which in turn has followed two main approaches. The first approach consists in developing strong cuts coming from facet-inducing inequalities associated with special structures in the input graph (for instance, cliques, odd holes, and webs) and searching for specialized separation techniques for these families of inequalities (an up to date list of references for this approach can be found in [17]). The second approach relies on general cut-generating procedures which, starting from a fractional solution, search for a violated inequality with no prespecified structure. Such procedures were either shown to be effective in practice [16,18] or to generate provably strong inequalities [26]. The main contribution of this work is general procedures that are both effective and generate inequalities that can be proved to be facet-inducing under quite general conditions.

We now discuss existing works following the second approach, i.e., procedures generating cuts with no prespecified structure. Mannino and Sassano [14] introduced in 1996 the idea of edge projections as a specialization of Lovász and Plummer's clique projection operation (see [12] Chapter 12.4; the clique projection is referred there as "clique reduction ". The authors use this notion to derive a polynomial time algorithm for the stable set problem in claw-free graphs). Many properties of edge projections are discussed in [14] and, based on these properties, a procedure computing an upper bound for the MSS problem is developed. This bound is then incorporated in a branch and bound scheme. Rossi and Smriglio take these ideas into an integer programming environment in [18], where a separation procedure based on edge projection is proposed. Finally, Rebennack et al. [16] extend the theory of edge projection by explaining the facetness properties of the inequalities obtained by this procedure. The authors give a branch and cut algorithm that uses edge projections as a separation tool, as well as several specific families of valid inequalities such as the odd hole inequalities (with a polynomial-time exact separation algorithm), the clique inequalities (with heuristic separation procedures), and mod-$\{2, 3, 5, 7\}$ cuts.

Rossi and Smriglio propose in [18] to employ a sequence of edge projection operations to reduce the original graph G and make it denser at the same time, allowing for a faster identification of clique inequalities on the reduced graph G'. This procedure iteratively removes and projects edges with certain properties, and heuristically finds violated rank inequalities (i.e., inequalities of the form $\sum_{v\in A} x_v \leq \alpha(G[A])$, where $A \subseteq V$ and $G[A]$ is the subgraph of G induced by A). A key step for achieving this is to be able to establish how $\alpha(G)$ is affected by these edge projections, or, in other words, how exactly $\alpha(G)$ relates to $\alpha(G')$. We aim at generalizing Rossi and Smriglio's procedure by projecting cliques instead of edges, so we also need to show how $\alpha(G)$ changes as a result of this operation. Our method allows thus to establish a more general relation between G and the graph resulting from the clique projection. We observe finally that clique projections for the MSS problem have also been studied in [21] for reducing instance sizes (not to generate cuts).

In this article we propose the use of clique projections as a general method for cutting plane generation for the MSS, along with new clique lifting operations that lead to stronger inequalities than those obtained with the edge projection method. The proposed method is able to generate both rank and weighted rank valid inequalities (to be defined below), by resorting to the general lifting operation introduced in [26]. This approach allows to produce cuts of a quite general nature, including cuts from the known families of valid inequalities for the MSS polytope. Based upon the projection and lifting operations, we give a separation procedure that departs from the usual template-based paradigm for generating cuts, and seeks to unify and generalize the separation procedures for the known cuts. In this sense, our main goal is to provide a more complete understanding of the maximum stable set polytope, which may help also in the solution of other combinatorial optimization problems. Experimental results are provided to validate the general procedure we propose.

This work is organized as follows. In Section 2 we define the MSS polytope $STAB(G)$, we define the operation of clique projection and we explore some basic facts on this operation. Section 3 introduces the crucial concept of *clique lifting*, based on the results in [26]. In Sections 4 and 5 we introduce our cut-generating method, by applying the lifting method presented in [26]. Finally, in Section 6 we present some computational experience on the DIMACS and randomly generated instances, which show that the method is competitive. The paper is closed with some concluding remarks in Section 7.

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