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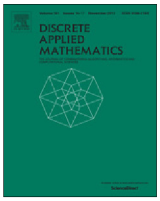

Generalized local branching heuristics and the capacitated ring tree problem

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ABSTRACT

In this paper we present a heuristic framework that is based on mathematical programming to solve network design problems. Our techniques combine local branching with locally exact refinements. In an iterative strategy an existing solution is refined by sequentially solving restricted mixed integer programs (MIPs) to optimality. These are obtained from the master problem MIP by limiting the number of variable flips for structured subsets of the binary edge variables which are selected based on the underlying network cost structure. We introduce generalized local branching cuts which enforce the latter using two parameters at the same time: the number of considered variables and the number of allowed variable flips.

Using this concept we develop an efficient algorithm for the capacitated ring tree problem (CRTP), a recent network design model for partially reliable capacitated networks that combines cycle and tree structures. Our implementation operates on top of an efficient branch and cut algorithm for the CRTP. The sets of refinement variables are deduced from single-ball network node clusters. We provide computational results and an extensive analysis of the algorithm for a set of literature instances. We show that the approach is capable of improving existing best results for the CRTP and outperforms the pure refinement or local branching approaches.

1. Motivation and contribution

Network design applications in telecommunications and transportation environments typically involve a large number of decision variables in suitable optimization models. Although exact algorithms are usually not applicable to medium and large size instances they have proven useful in heuristic frameworks, also referred to as *matheuristics* [19]. This class of heuristics combines mathematical programming concepts and classical (meta-)heuristic paradigms.

Integer programming based *refinement algorithms* have been successfully applied to complex network optimization problems (e.g. [6,2,4,13,20]) as well as other classes of challenging combinatorial optimization problems (e.g. [19,16]). These methods typically incorporate an exact mathematical programming based approach that is applied to a local improvement model for an existing solution. They get more effective with increasing complexity of the underlying problem structure [2,13]. Due to the limited computational efficiency of the exact method that is used to carry out the refinements, the mentioned techniques are in fact only effective locally on small-sized substructures. Commonly, random, multi-start or contraction based perturbation mechanisms are added to (partially) overcome local optimality.

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In this work we suggest an approach that aims at increasing the number of decisions considered for local refinement. This is achieved by bounding the scale of modification in terms of binary variable flips in return. The basic idea of considering neighboring solutions within a certain Hamming distance is known as *local branching* and has been introduced as a polishing procedure in general mixed integer programs (MIPs) in [8]. Several highly efficient heuristics for various combinatorial optimization problems successfully incorporate this concept (e.g. [21,18,22]). In [17], a refinement method for general binary MIPs is presented that is closely related to the relaxation induced neighborhood search method (RINS) [5]. Both approaches use information from the solution of the linear relaxation of the MIP to select variables and therewith define neighborhoods that are explored in an exact fashion. In contrast to these works, our approach does not use fractional solutions to guide the local search. It is solely based on the structural exploration of neighborhoods that are derived from the incumbent solution network and the underlying edge costs. The corresponding simultaneous combination of subsets of binary variables of different sizes with suitable Hamming bounds provides a complete algorithmic framework. We formalize the iterative construction of refinement models by describing new *generalized local branching cuts* which are added to the master program before solving these extended MIPs to optimality. Herewith, we are able to arbitrarily increase the local area that is considered for refinement by adequately limiting the allowed variable flips.

We show that the sketched ideas can be turned into an effective algorithm for capacitated network design. More precisely, we devise an efficient heuristic for the capacitated ring tree problem (CRTP). The CRTP was introduced in an earlier paper of Hill and Voß [14]. It combines ring based models such as the classic traveling salesman problem (TSP) with tree based models such as the Steiner tree problem (STP) under capacity constraints. Heuristics and exact algorithms for the CRTP are discussed in [10,14] and [15]. A facility location variant of the problem is considered in [1] and algorithms for bi-objective reliability-oriented variants are elaborated in [12]. Even though the CRTP can be broadly applied as it generalizes several prominent network design problems, our techniques can be transferred to related network design problems with reasonably low effort. More generally, we suggest that our main ideas can be used to solve a variety of discrete, or even (partially) continuous, optimization problems. The main contributions of this work are

- the development of a generic framework for heuristic network design based on a generalized local branching, combining local branching and integer programming based refinement techniques, and
- the design and analysis of an efficient heuristic algorithm for the CRTP incorporating these concepts which is able to find new best solutions for literature instances.

Section 2 contains a formal description of the CRTP along with the MIP formulation used in our algorithm. After the presentation of the generic local branching based refinement technique in Section 3 we develop the heuristic algorithm for the CRTP in Section 4. In Section 5 we provide the improved results for literature instances and computationally compare different configurations of our method. We close the paper with conclusions in Section 6.

2. The capacitated ring tree problem

In this section, we formally define the considered network design model, describe an application, and provide a mathematical model based on integer programming.

2.1. Problem definition

The *capacitated ring tree problem* (CRTP) [14] generalizes several classical ring and tree based network design models. The base topology in a solution network is the *ring tree*, defined as a graph consisting of a cycle $\mathcal{C}$ and a set of node disjoint trees $\mathcal{T}$, each of them intersecting with $\mathcal{C}$ in exactly one node. A ring tree reduces to a pure tree if $|\mathcal{C}| = 1$, and to a cycle if $\mathcal{T} = \emptyset$. To simplify our description, we say that a *ring tree star, centered in d*, is a graph obtained by a union of ring trees that intersect in d such that for each of these ring trees $\mathcal{Q}$,

- d is a leaf in $\mathcal{Q}$ if $\mathcal{Q}$ is a pure tree, and
- d is a cycle node of degree 2 in $\mathcal{Q}$, otherwise.

Fig. 1 depicts three characteristic ring tree stars.

In the CRTP, we are given customer nodes $U = U_1 \dot{\cup} U_2$, Steiner nodes W, a depot node d, and capacity bounds $m, q \in \mathbb{N}$. Denote the set of all nodes as $V = U \dot{\cup} W \dot{\cup} \{d\}$ and the set of potential edges between two nodes as E. Let c_e be the given cost for the installation of an edge $e \in E$. A ring tree star $\mathcal{N}$ on nodes from V, centered in d, is a solution for the CRTP if

(i) each customer node in U is contained in $\mathcal{N}$,
(ii) each customer node in U_2 is on a cycle in $\mathcal{N}$,
(iii) $\mathcal{N}$ is composed of at most m ring trees, and
(iv) each ring tree in $\mathcal{N}$ contains at most q customers.

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