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Hamiltonian properties of some compound networks

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ABSTRACT

Given two regular graphs G and H, the *compound graph* of G and H is constructed by replacing each vertex of G by a copy of H and replacing each link of G by a link which connects corresponding two copies of H. Let $DV(m, d, n)$ be the compound networks of the disc-ring graph $D(m, d)$ and the hypercube-like graphs HL_n, and $DH(m, d, n)$ be the compound networks of $D(m, d)$ and $\mathcal{H}_n$ which is the set of all $(n-2)$-fault Hamiltonian and $(n-3)$-fault Hamiltonian-connected graphs in HL_n. We obtain that every graph in $DV(m, d, n)$ is Hamiltonian which improves the known results that the *DTcube*, the *DLcube* and the *DCcube* are Hamiltonian obtained by Hung [Theoret. Comput. Sci. 498 (2013) 28-45]. Furthermore, we derive that $DH(m, d, n)$ is $(n-1)$-edge-fault Hamiltonian. As corollaries, the $(n-1)$-edge-fault Hamiltonicity of the $DRHL_n$ including the $DT(m, d, n)$ and the $DC(m, d, n)$ is obtained. Moreover, the $(n-1)$-edge-fault Hamiltonicity of $DH(m, d, n)$ is optimal.

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1. Introduction

An interconnection network can be characterized by a loop-less undirected graph $G = (V, E)$, where V is the set of processors and E is the set of communication links. In this paper, we use graphs and networks interchangeably. To an interconnection network, desirable properties, such as symmetry, small diameter, embedding capabilities, Hamiltonicity, fault-tolerance, are necessary. Among a number of interconnection networks, the hypercube-like graph is well designed with many favorable properties. Many hypercube-like graphs have been studied such as hypercube [11], twisted cube [6,14], and crossed cube [2,3].

For the Hamiltonicity of a graph, there is no nontrivial necessary and sufficient condition for a graph to be Hamiltonian, and that finding a Hamiltonian cycle or path is NP-Complete [4,8]. Therefore, considerable studies on Hamiltonicity focus on different networks. For n-dimensional hypercube-like interconnection networks HL_n, it has been proved that every bipartite graph in HL_n is Hamiltonian-laceble and every non-bipartite graph in HL_n is Hamiltonian-connected. It is nearly impossible to promise multiprocessor systems without defects in the operation. Suppose faults might happen on any edges in an interconnection network without any restriction, many researchers consider the random fault model for the Hamiltonicity of various networks. It is limited by the minimal degree of a graph G denoted by $\delta(G)$, so the optimal result is that a network G is $(\delta(G)-2)$-edge-fault Hamiltonian.

Let HL_n be the hypercube-like graphs and $\mathcal{H}_n$ be the set of graphs in HL_n which are $(n-2)$-fault Hamiltonian and $(n-3)$-fault Hamiltonian-connected. Hung provided a network topology called disc-ring graphs $D(m, d)$, with many nice topological properties. Let $DV(m, d, n)$ be the compound networks, called *DVcube*, of the disc-ring graph $D(m, d)$ and HL_n. The family of graphs $DH(m, d, n)$ is the compound networks of $D(m, d)$ and $\mathcal{H}_n$. Hung [7] presented some specific families of *DVcube* including the *DQcube*, the *DTcube*, the *DLcube* and the *DCcube* and gave Hamiltonian properties for some of them.

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We generalized these results and obtained that every graph in $DV(m, d, n)$ is Hamiltonian. Furthermore, we derive that $DH(m, d, n)$ is $(n-1)$-edge-fault Hamiltonian. Those are Theorems 1.1 and 1.2.

Theorem 1.1. *Every graph in DVcube $DV(m, d, n)$ is Hamiltonian.*

Theorem 1.2. *Let G be any graph in DHcube $DH(m, d, n)$ with $n \geq 2$, then G is $(n-1)$-edge-fault Hamiltonian.*

Since the *DQcube*, the *DTcube*, the *DLcube* and the *DCcube* are specific families of *DVcube*, some Hamiltonicities of them obtained by Hung [7] are the corollaries of Theorem 1.1.

Corollary 1.3. *The DQcube $DQ(m, d, n)$ which is the compound graph with the disc-ring graph and the hypercube Q_n is Hamiltonian.*

Corollary 1.4 ([7])**.** *The DTcube $DT(m, d, n)$ which is the compound graph with the disc-ring graph and the twisted cube TQ_n is Hamiltonian.*

Corollary 1.5 ([7])**.** *The DLcube $DL(m, d, n)$ which is the compound graph with the disc-ring graph and the locally twisted cube LTQ_n is Hamiltonian.*

Corollary 1.6 ([7])**.** *The DCcube $DC(m, d, n)$ which is the compound graph with the disc-ring graph and the crossed cube CQ_n is Hamiltonian.*

Since the $DRHL_n$ including the $DT(m, d, n)$ and the $DC(m, d, n)$ is the subset of $DH(m, d, n)$, thus the following corollaries are obtained from Theorem 1.2 directly.

Corollary 1.7. *The compound graph, denoted by $DRHL_n$, with the disc-ring graph and the restricted hypercube-like graph RHL_n is $(n-1)$-edge-fault Hamiltonian.*

Corollary 1.8. *The DTcube $DT(m, d, n)$ which is the compound graph with the disc-ring graph and the twisted cube TQ_n is $(n-1)$-edge-fault Hamiltonian.*

Corollary 1.9. *The DCcube $DC(m, d, n)$ which is the compound graph with the disc-ring graph and the crossed cube CQ_n is $(n-1)$-edge-fault Hamiltonian.*

The remainders of this paper are organized as follows. Section 2 introduces necessary definitions and notations. Hamiltonicity of *DVcube*, that is, the proof of Theorem 1.1 is given in Section 3. Edge-fault-tolerant Hamiltonicity of *DHcube*, that is, the proof of Theorem 1.2 is shown in Section 4. Concluding remarks are given in Section 5.

2. Definitions and preliminaries

Let $G = (V, E)$ be a simple undirected graph. Two vertices v_1, v_2 in V are said to be *adjacent* if and only if $(v_1, v_2) \in E$ and v_1, v_2 are said to be *incident* with the edge (v_1, v_2), and v_1, v_2 are called *ends* of the edge (v_1, v_2). Two edges which are incident with a common vertex are also said to be *adjacent*. The *neighbor* of a vertex u is a vertex adjacent to u in G and the *neighborhood* of a vertex u is a set of all neighbors of u in G, denoted by $N_G(u)$. The cardinality $|N_G(u)|$ represents the *degree* of u in G, denoted by $d_G(u)$ (or simply $d(u)$), $\delta(G)$ is the *minimum degree* of G. Suppose that V' is a nonempty subset of V and E' is a nonempty subset of E, then $G - V'$ is the subgraph obtained from G by deleting the vertices in V' together with their incident edges. If $V' = \{v\}$, we write $G - v$ for $G - \{v\}$; and $G - E'$ is the subgraph obtained from G by deleting the edges in E'. Similarly, the graph obtained from G by adding a set of edges in E' is denoted by $G + E'$. If $E' = \{e\}$, we write $G - e$ and $G + e$ instead of $G - \{e\}$ and $G + \{e\}$. A *path* $P = (v_1, v_2, \ldots, v_k)$ for $k \geq 2$ in G is a sequence of distinct vertices such that any two consecutive vertices are adjacent. The two vertices v_1 and v_k are the *ends* of a path P, and P is also called a (v_1, v_k)-*path*. A path $P = (v_1, v_2, \ldots, v_k)$ forms a *cycle* C if $v_1 = v_k$ and $k \geq 3$. For a graph $G = (V, E)$, if the vertex set V can be partitioned into two subsets X and Y such that each edge has one end in X and one end in Y, then G is a bipartite graph denoted by $G(X, Y)$.

A graph $G = (V, E)$ is *Hamiltonian* if there exists a *Hamiltonian cycle* which is a cycle containing all the vertices of the graph G and it is *Hamiltonian-connected* if there exists a *Hamiltonian path* which is a path containing all the vertices of the graph G connecting any pair of vertices. If G is a bipartite graph, it is *Hamiltonian-laceable* if there exists a Hamiltonian path connecting any pair of vertices in different parts. We call G is *k-fault Hamiltonian* (resp. *k-fault Hamiltonian-connected*) if there exists a Hamiltonian cycle (resp. if each pair of vertices can be joined by a Hamiltonian path) in $G \setminus F$ for any set F of elements (vertices and/or edges) with $|F| \leq k$. For any deleted set $F \subseteq E$ with $|F| \leq k$, then G is *k-edge-fault Hamiltonian* if $G \setminus F$ is Hamiltonian.

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