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Hamiltonian laceability in hypercubes with faulty edges[☆]

Fan Wang[a,*], Heping Zhang[b]

[a] School of Sciences, Nanchang University, Nanchang, Jiangxi 330000, PR China

[b] School of Mathematics and Statistics, Lanzhou University, Lanzhou, Gansu 730000, PR China

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ABSTRACT

It is useful to consider faulty networks because node faults or link faults may occur in networks. In this paper, we investigate hamiltonian properties of conditional faulty hypercubes. Let F be a set of faulty edges in hypercube Q_n with $n \geq 4$ and $|F| \leq 3n - 11$. We prove that there still exists a hamiltonian path in $Q_n - F$ joining any two vertices of different partite sets if the following two constraints are satisfied: (1) the degree of every vertex in $Q_n - F$ is at least 2, and (2) there is at most one vertex with degree 2 in $Q_n - F$.

1. Introduction

A topological structure of an interconnection network can be modeled by a graph $G = (V(G), E(G))$, where the vertex set $V(G)$ represents the set of processors and the edge set $E(G)$ represents the set of links joining processors. Investigating structures of G is essential to design a suitable topology of interconnection network. Structures of paths and cycles, which are fundamental topologies for parallel and distributed processing, are suitable for local area networks and for the development of simple parallel algorithms with low communication cost [1].

A path (respectively, cycle) in a graph G is a *hamiltonian path* (respectively, *hamiltonian cycle*) if every vertex in G appears exactly once in the path (respectively, cycle). A graph is *hamiltonian* if it has a hamiltonian cycle. A graph is *hamiltonian connected* if there exists a hamiltonian path joining any two distinct vertices in it. A graph $G = (V_0 \cup V_1, E)$ is *bipartite* if $V(G)$ is the union of two disjoint sets V_0 and V_1 such that each edge consists of one vertex from each set. As the hamiltonicity of a graph G is concerned, it is an important issue to investigate if G is hamiltonian or hamiltonian connected. However, any hamiltonian bipartite graph $G = (V_0 \cup V_1, E)$ satisfies $|V_0| = |V_1|$. Since the vertices in V_0 and V_1 alternate in each path, all hamiltonian bipartite graphs are not hamiltonian connected. Simmons [17] introduces the concept of hamiltonian laceability for those hamiltonian bipartite graphs. A hamiltonian bipartite graph $G = (V_0 \cup V_1, E)$ is *hamiltonian laceable* if there is a hamiltonian path joining any two vertices x and y with $x \in V_0$ and $y \in V_1$.

The n-dimensional hypercube, denoted by Q_n, is one of the most popular and efficient interconnection networks. It is well known that Q_n is hamiltonian for every $n \geq 2$ [12], and Q_n is hamiltonian laceable for every $n \geq 1$ [13]. Fault-tolerance is an important index of the stability of the network. It is useful to consider faulty networks because node faults or link faults may occur in networks [3,9,10,14,18,20].

Given a set F of faulty edges in the hypercube Q_n and a pair of vertices x, y, is there a hamiltonian cycle or a hamiltonian path joining x and y in $Q_n - F$ [8]? Chan and Lee [4] showed for $n \geq 3$ that any Q_n, where each vertex is incident with at least two nonfaulty edges, has a fault-free hamiltonian cycle even if it has up to $(2n - 5)$ edge faults. This result is optimal since a hypercube which contains the graph depicted in Fig. 1 as a subgraph has no fault-free hamiltonian cycle.

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* Corresponding author.
E-mail addresses: wangfan@ncu.edu.cn (F. Wang), zhanghp@lzu.edu.cn (H. Zhang).

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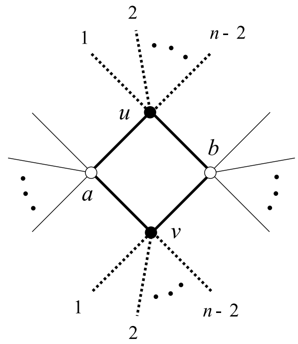

Fig. 1. The forbidden subgraph in which faulty edges are represented by dashed lines.

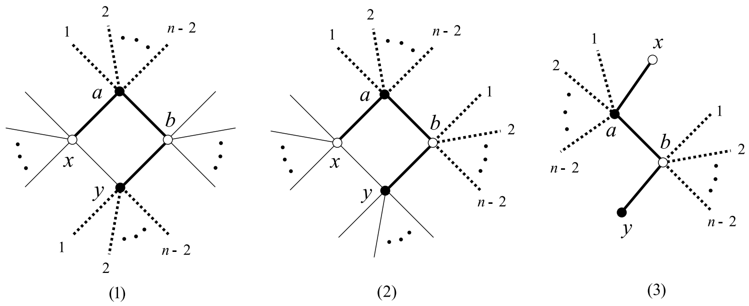

Fig. 2. The forbidden subgraphs in which faulty edges are represented by dashed lines.

Liu and Wang [16] generalized the above result.

Theorem 1.1 ([16]). *Let F be a set of faulty edges in Q_n with $n \geq 5$ and $|F| \leq 3n-8$. Then there exists a hamiltonian cycle in $Q_n - F$ under the following two constraints:*
(1) *the degree of every vertex in $Q_n - F$ is at least 2, and*
(2) *there do not exist a pair of nonadjacent vertices in a 4-cycle whose degrees are both two.*

Tsai [19] considered the problem of hamiltonian paths in the faulty hypercube, and obtained the following result.

Theorem 1.2 ([19]). *For $n \geq 3$, let F be a set of faulty edges in Q_n with $|F| \leq 2n-5$. If the degree of every vertex in $Q_n - F$ is at least 2, then there exists a hamiltonian path in $Q_n - F$ joining any two vertices of different partite sets.*

Observe that if Q_n has a subgraph as depicted in Fig. 2(1)–(3), then there exists no hamiltonian path joining vertices x and y in $Q_n - F$.

In this paper, we show that the number of faulty edges can be extended to $3n-11$ for $n \geq 4$ such that there still exists a hamiltonian path in $Q_n - F$ joining any two vertices of different partite sets under the following two constraints:
(1) the degree of every vertex in $Q_n - F$ is at least 2, and
(2) there is at most one vertex with degree 2 in $Q_n - F$.

The rest of the paper is organized as follows. In Section 3 we present the proof of the main result. Section 2 presents some results which are applied in the proof of the main result.

2. Definitions and preliminaries

Terminology and notation used in this paper but undefined below can be found in [2]. Let G be a graph. For a set $F \subseteq E(G)$, let $G - F$ denote the resulting graph after removing all edges in F from G. For a set $S \subseteq V(G)$, let $G - S$ denote the graph removing all vertices in S and all the edges incident with S from G. Let H and H' be two subgraphs of G. We use $H + H'$ to denote the graph with the vertex set $V(H) \cup V(H')$ and edge set $E(H) \cup E(H')$. For $F \subseteq E(G)$, let $V(F)$ denote the set of vertices incident with F. We use $H + F$ to denote the graph with the vertex set $V(H) \cup V(F)$ and edge set $E(H) \cup F$. When $S = \{x\}$ and $F = \{e\}$, we simply write $G - S$, $G - F$, $H + F$ and $V(F)$ as $G - x$, $G - e$, $H + e$ and $V(e)$.

The *n-dimensional hypercube* Q_n is a graph whose vertex set consists of all binary strings of length n, i.e., $V(Q_n) = \{u : u = u^1 \cdots u^n$ and $u^i \in \{0, 1\}$ for every $i \in \{1, \ldots, n\}\}$, with two vertices being adjacent whenever the corresponding strings differ in just one direction.

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