Constructing day-balanced round-robin tournaments with partitions

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ABSTRACT

Considering several notions of balance, for almost all admissible values of p and n we provide constructions for such balanced round-robin tournaments with p parts of n teams each, where each team plays λ_1 games against teams from its own part, and λ_2 games against teams from other parts.

When n is even, we also provide constructions for such balanced round-robin tournaments in the setting where each day of the tournament a part receives a bye.

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1. Introduction

Sports scheduling problems have been studied extensively for many decades. One particular reason that made this field so interesting is that it brings graph theory and combinatorial design theory together with operations research. (A survey on how to design tournaments with certain desirable properties from a combinatorial design theory perspective using latin squares can be found in [14].)

A round-robin tournament (for the purposes of this paper) is a tournament where on each day of the tournament each team plays exactly one game. It is often desirable to have some notions of balance imposed on round-robin tournaments to meet certain fairness criteria of the schedules. In this context, the much celebrated early work by de Werra [5–8], where he proved that for each k every bipartite graph has a k-edge-coloring that satisfies several notions of balance at the same time, has become the key for attacking such scheduling problems using delicate edge-coloring techniques.

The literature related to the design of round-robin tournaments is too broad to include here, instead we note some inspiring papers that focus on round-robin tournaments with partitions, as this is the focus of the present paper. Indeed, round-robin tournaments with partitions are of particular value because different parts of a tournament can usually be thought of as different strength levels, subleagues, conferences, divisions etc., which provides a vast area of applications of such tournaments. (In order to define our results most inclusively, in this paper we will refer to "parts" or "partitions" rather than any one of the aforementioned potential settings.) An early paper by Straley [17] on scheduling a sports league that is partitioned into conferences appeared in 1983. Briskorn [2], and Briskorn and Knust [3] considered partitioned tournaments from the perspective of strength groups. Burrows and Tuffley [4] considered tournaments with two divisions of $2n$ and $2n+2$ teams respectively, where the $2n$ clubs represented by a team in the first division are also represented by a team in the second division, and they settled the problem of maximizing common fixtures. Two papers by de Werra, Ekim and Raess [9], and by Geinoz, Ekim and de Werra [11] both of which concern balancing games in stadiums also deserve attention in this context:

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even though the focus of these aforementioned two papers is not partitioned tournaments, they are relevant in that they use the idea of partitioning a tournament into subtournaments.

In this paper we provide constructions for balanced round-robin tournaments with p parts of n teams each, where each team plays λ_1 games against teams from its own part, and λ_2 games against teams from other parts. Such tournaments arise naturally in the scheduling of many well known sports leagues. We give two particularly interesting examples. In NBA (National Basketball Association) Season 1967–1968 there were a total of 12 teams divided equally into two conferences, where each team played 8 times against teams from the same conference, and 7 times against teams from the other conference; totaling up to 82 games in the season. XFL was an American football league that played only one season in 2001. There were two divisions (Eastern and Western) consisting of 4 teams each, where each team played twice against teams of the same division, and once against teams of the other division; thus totaling up to 10 games in the season.

The particular natural notions of balance that we require a partitioned tournament to satisfy simultaneously are defined next. A game played between some team from part i and some other team from part j will be referred to as an *i-j* game. For any parts i, j, k, l $(i \neq j, k \neq l)$ and for any two days of the tournament x, y (possibly $x = y$),

   (i) the number of *i-j* games on day x differs from the number of *i-j* games on day y by at most 1,
  (ii) the number of *i-j* games on day x differs from the number of *k-l* games on day y by at most 1,
 (iii) the number of *i-i* games on day x differs from the number of *i-i* games on day y by at most 1, and
 (iv) the number of *i-i* games on day x differs from the number of *k-k* games on day y by at most 1.

Note that in the setting where there are the same number of *i-j* games $(i \neq j)$ as the number of *k-l* games $(k \neq l)$ in a tournament (as is the case in this paper), condition (ii) follows from condition (i). Similarly, as in this paper, if there are the same number of *i-i* games as the number of *k-k* games in a tournament, condition (iv) follows from condition (iii).

It would be monotonous to have too many games between teams from the same two parts (say, conferences) on a certain day of the tournament, but few or no such games on some other day, so the balance condition (i) is very plausible. By similar insight it can be justified that the conditions (ii), (iii) and (iv) can equally be desirable for round-robin tournaments with partitions. The merit in requiring some of the aforementioned balance notions is appreciated when the tournament schedule is examined for 2014 FIFA World Cup that took place in Brazil. The group stage of the World Cup consisted of 32 teams. These teams were initially divided into 4 pots with 8 teams each (based on the geographic regions and the strength of the teams) as follows:

① Host/Top 7	② Africa/S. America/Italy	③ Asia/N. America	④ Europe
Argentina	Algeria	Australia	Bosn. & Herz.
Belgium	Cameroon	Costa Rica	Croatia
Brazil	Chile	Honduras	England
Colombia	Ecuador	Iran	France
Germany	Ghana	Japan	Greece
Spain	Italy	Mexico	Netherlands
Switzerland	Ivory Coast	South Korea	Portugal
Uruguay	Nigeria	United States	Russia

Eight groups of 4 teams were then formed with exactly one team from each pot as follows.

Brazil	Spain	Colombia	Uruguay
Cameroon	Chile	Ivory Coast	Italy
Mexico	Australia	Japan	Costa Rica
Croatia	Netherlands	Greece	England

Switzerland	Argentina	Germany	Belgium
Ecuador	Nigeria	Ghana	Algeria
Honduras	Iran	United States	South Korea
France	Bosn. & Herz.	Portugal	Russia

In the group stage, each team played one game against the teams in the same group and zero games against the teams from the other groups. At this stage, it would have been beneficial for the broadcasters and the viewers if the tournament schedule could avoid having too many games between teams from the same two pots, say Asia/N. America and Europe, on one day of the tournament, in order to provide the viewers with a wider range of encounters of soccer styles of different regions in the world on each day of the tournament. The essence of this extra requirement is captured by condition (i).

In the actual program of World Cup 2014, the distribution of the games in the group stage was as given below:

Day 1: two ①–② games, one ①–③ game, five ①–④ games
            five ②–③ games, one ②–④ game, two ③–④ games
Day 2: three ①–② games, two ①–③ games, three ①–④ games
            three ②–③ games, two ②–④ games, three ③–④ games
Day 3: three ①–② games, five ①–③ games, zero ①–④ games
            zero ②–③ games, five ②–④ games, three ③–④ games.