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Lower bounds for the Event Scheduling Problem with Consumption and Production of Resources

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ABSTRACT

The Event Scheduling Problem with Consumption and Production of Resources (ESPCPR) is a general scheduling problem where the availability of a resource is depleted and replenished at the occurrence times of a set of events. This problem is an extension of the Resource Constrained Project Scheduling Problem (RCPSP) where activities are replaced by events, which have to be scheduled subject to generalized precedence relations. The aim of this paper is to show the connections between ESPCPR and classical scheduling problems in order to bring new lower bounds for ESPCPR. We start by recalling some similar models in literature such as the Project Scheduling with Inventory Constraints and the Financing Problem. Subsequently, we describe how to model classical scheduling problems using ESPCPR. Finally, we propose four lower bounds for this problem. The obtained computational results confirm their effectiveness.

1. Introduction

The Resource Constrained Project Scheduling Problem (RCPSP) is the basic problem type in project scheduling. It is without doubt the most widely studied scheduling problem in literature, and it has resulted in an overwhelming amount of papers with solution procedures devoted to it. In this problem, non-preemptive activities requiring renewable resources have to be scheduled subject to precedence constraints so as to minimize the makespan. Renewable resources are allocated to activities at their starting time and released at their completion time. On the contrary, a *nonrenewable* resource is produced or consumed by an activity at its starting time only. Money is an example of nonrenewable resource for which Carlier and Rinnooy Kan introduced the financing problem [7].

In this paper we address the Event Scheduling Problem with Consumption and Production of Resources (ESPCPR). ESPCPR is a general scheduling problem where the availability of resources is depleted and replenished [3]. An instance of ESPCPR consists of events, nonrenewable resources and generalized precedence constraints between pairs of events. Each event produces or consumes some units of resources at its occurrence time. The objective is to build a schedule that satisfies the precedence and resource constraints and minimizes the makespan.

ESPCPR is a generalization of RCPSP where activities requiring renewable resources are replaced by events consuming or producing nonrenewable resources. Some other authors have worked on models similar to ESPCPR. We can quote the works of Neumann and Schwindt [15] and of Laborie [12]. Neumann and Schwindt formalized the Project Scheduling Problem with Inventory Constraints, where the availability of each resource is at any time bounded from below and from above. To solve this problem, they proposed a branch-and-bound algorithm with a filtered beam search heuristic. Laborie [12] introduced

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the concept of a Resource Temporal Network (RTN). He proposed a constraint propagation algorithm to solve the problem. Koné et al. [11] worked on the RCPSP with Consumption and Production of Resources (RCPSP/CPR). The particularity of this extension of RCPSP is that, in addition to renewable resources considered in the basic version, it also involves nonrenewable resources which can be consumed (or not) at the starting time of an activity in a certain amount and/or then produced in another amount at the completion time of this activity. To solve this problem, Koné et al. proposed four mixed integer linear programming models for RCPSP/CPR. ESPCPR coincides with the problem considered by [15,12] where no upper bound on the resource availability is prescribed. Note that the relationship between ESPCPR and RCPSP/CPR will be elaborated in Section 3.2.

In a recent paper Carlier et al. [3] have generalized tools introduced for the RCPSP to the ESPCPR. For instance schedules can be built by using list algorithms. In this work, we have also been inspired by previous works on scheduling problems with renewable resource such as the Cumulative Scheduling Problem to develop new lower bounds for ESPCPR.

Lower bounds have been proposed for models similar to the ESPCPR. Neumann and Schwindt [15] introduced two lower bounds for the Project Scheduling Problem with Inventory Constraints. One is a critical path based lower bound and the other one is similar to the Shifting Algorithm which was introduced for the Financing Problem [2]. Selle [17] proposed a lower bound based on a time-indexed mixed-integer programming formulation and a Lagrangian relaxation of the resource constraints.

The purpose of this paper is to introduce four lower bounds for ESPCPR. Two of them are based on the extraction of a generalized Cumulative Scheduling Problem, combined with an adapted version of Jackson's Pseudo-Preemptive Schedule [6] and the concept of energetic reasoning. Two further lower bounds respectively result from applying Carlier and Rinnooy Kan's Shifting Algorithm to a Financing Problem and iteratively testing the feasibility of associated network flow problems in a dichotomic search method.

The remaining part of this paper is structured as follows. In Section 2 we present some terminology dedicated to basic concepts and formulate the ESPCPR. In Section 3 we introduce some existing models in literature with production and consumption of resources. In Section 4 we present the Jackson's Pseudo-Preemptive Schedule and its different adaptations. In Section 5 we present our four new lower bounds. In Section 6 we report on experimental results and comment on the practical efficiency of our lower bounds, and finally we conclude this work in Section 7.

2. Event scheduling problem with consumption and production of resources

An instance $I = (X, U, a, v)$ of ESPCPR is defined by a set $X = \{0, \ldots, n+1\}$ of events, and a set U of generalized precedence constraints which express relations of start-to-start between pairs of events. By convention, the two events 0 and $n+1$ are added to respectively define the start and the end of the schedule. The number of resource units produced or consumed by event i is defined by a_i, where a_0 corresponds to the initial resource units of the project. If $a_i < 0$, then event i consumes $|a_i|$ resource units, whereas if $a_i \geq 0$, it produces $|a_i|$ resource units. In case of multiple resources, K is the set of nonrenewable resources and a_i^k defines the quantity of resource k produced or consumed by event i, where $k \in K$.

A *schedule* S on event set X is a function assigning an occurrence time $S(i)$ to each event $i \in X$. The time lag between event i and event j is equal to v_{ij}. If $v_{ij} \geq 0$, then event j cannot occur before time $S(i) + |v_{ij}|$. If $v_{ij} < 0$, then event i has to occur no later than time $S(j) + |v_{ij}|$. The makespan of a schedule S can be computed as $C_{\max} = S(n+1)$. A schedule is feasible if it satisfies the generalized precedence constraints

$$S(i) + v_{ij} \leq S(j) \quad \forall (i, j) \in U$$

and the resource constraints

$$\sum_{i \in X(S,t)} a_i^k \geq 0 \quad \forall k \in K, \ \forall t \in [0 \ldots H]$$

where $X(S, t) = \{i \in X \mid S(i) \leq t\}$ is the set of events which have occurred by time $t \geq 0$, and H is some given upper bound on the makespan, which means that all events have to occur no latter than time H. An optimal schedule is a feasible schedule which minimizes the makespan. So, the Event Scheduling Problem with Consumption and Production of Resources can be formulated as follows:

$$
\begin{array}{llll}
\text{Minimize} & S(n+1) & & (1) \\
\text{subject to} & \sum_{i \in X(S,t)} a_i^k \geq 0 & \forall k \in K, \forall t \in [0 \ldots H] & (2) \\
& S(i) + v_{ij} \leq S(j) & \forall (i, j) \in U & (3) \\
& S(i) \geq 0 & \forall i \in X \setminus \{0\} & (4) \\
& S(0) = 0 & . & (5)
\end{array}
$$

A simple example of ESPCPR instance consists of seven events ($X = \{0, 1, 2, 3, 4, 5, 6\}$) with their production or consumption on a single resource $[+3, +2, -3, -3, +4, -1, 0]$, and having the following generalized precedences: $v_{13} = 3$

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