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Scheduling a single machine with parallel batching to minimize makespan and total rejection cost[☆]

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ABSTRACT

We consider the problem of scheduling a set of n jobs on a single machine with parallel batching and with rejection being allowed. Two bi-criteria problems are considered: (a) minimize the makespan subject to the constraint that the total rejection cost does not exceed a given threshold, and (b) minimize the total rejection cost subject to the constraint that the makespan does not exceed a given threshold. For the case of a batching machine with infinite capacity (i.e., the batch size allowed on the machine is larger than or equal to the number of jobs), we assume that the jobs have release dates. We present an $O(n^2)$-time 2-approximation algorithm for problem (a) and, in addition, we present dynamic programming algorithms and fully polynomial-time approximation schemes for both problems (a) and (b). For the case of a batching machine with finite capacity (i.e., the batch size allowed on the machine is less than the number of jobs), we assume that the jobs have identical release dates. We propose approximation algorithms for (a) and present dynamic programming algorithms and fully polynomial-time approximation schemes for both problems (a) and (b).

1. Introduction

Consider a single batching machine and n jobs with processing times $p_1, p_2, \ldots, p_n$, release dates $r_1, r_2, \ldots, r_n$, and weights $w_1, w_2, \ldots, w_n$. Let $\mathcal{J}$ denote the set of n jobs. The batching machine can process up to b jobs simultaneously. Whenever a set of jobs are processed on the machine in batch mode, the jobs in the set have to start at the same time and the jobs in the set can only be taken off the machine when the longest job in the set has completed its processing. The batching machine may allow for an unlimited batch size, i.e., $b \geq n$, or it may have a limited capacity, i.e., $b < n$. This type of batching is in the literature often referred to as parallel batching; this is in contrast to serial batching, where jobs of a batch are processed on the machine one after another with a setup time being required when the first job of a batch is about to

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start. In this paper, we will only consider parallel batching. The decision maker may decide not to process all n jobs. She may decide to reject job j and not process this job at all, but she then incurs a rejection penalty w_j.

The decision maker is interested in two objectives, namely the makespan $C_{\max}$ and the total rejection cost RC of the jobs rejected, i.e.,

$$RC = \sum_{j \in \mathcal{R}} w_j,$$

where $\mathcal{R}$ denotes the set of jobs being rejected. Following the problem classification schemes developed by T'kindt and Billaut [1] and by Shabtay et al. [2], a distinction is made in what follows between several types of scheduling problems with rejection. Typically, in a scheduling problem with rejection, the problem has, besides the total rejection cost, a second objective which in our case is the makespan $C_{\max}$. In the standard framework presented in the literature problem P1 would refer to the off-line scheduling problem with the objective of minimizing $C_{\max} + RC$. Problem P2 would refer to the problem in which the makespan $C_{\max}$ has to be minimized subject to the constraint that the total rejection cost RC does not exceed a given threshold R. Problem P3 would refer to the scheduling problem in which the total rejection cost RC is minimized subject to the makespan $C_{\max}$ being less than or equal to a given threshold H. Problem P4 refers to the problem in which for each Pareto-optimal point a Pareto-optimal solution is identified.

In this paper we focus on problems P2 and P3.

P2: Minimize the makespan under the condition that the total rejection cost does not exceed a given threshold R_0.

P3: Minimize the total rejection cost under the condition that the makespan does not exceed a given threshold H_0.

There is already a fairly extensive literature on batch scheduling (including a survey paper) and a fairly extensive literature on scheduling with rejection (including a survey paper as well). However, the literature that focuses on batch scheduling and rejection at the same time is still somewhat limited.

A number of papers have appeared on the scheduling of a single batching machine, e.g., Brucker et al. [3], Cheng et al. [4], Lee and Uzsoy [5], Liu and Yu [6], and Liu et al. [7]. Potts and Kovalyov [8] presented a survey on batch scheduling.

Single machine scheduling with rejection has also received a fair amount of attention in the literature, see Zhang et al. [9,10]. Lu et al. [11] dealt with a single-machine scheduling with rejection to minimize the makespan in an online environment. Shabtay et al. [2] presented a survey on scheduling models with rejection.

Single machine scheduling problems with parallel batching and rejection have been studied in most cases with the objective function being the sum of a scheduling objective and the total rejection cost. As for the problem with the makespan being a part of the objective, Lu et al. [12] considered a single batching machine with unlimited capacity ($b \geq n$), with release dates and the objective being the minimization of the sum of the makespan and the total rejection cost. They proved that the problem is binary NP-hard and developed a Fully Polynomial Time Approximation Scheme (FPTAS). Feng and Liu [13] proposed a polynomial time algorithm when the number of release times is fixed, and presented a Polynomial Time Approximation Scheme (PTAS) for this problem for the general case. For a single batching machine with limited capacity ($b < n$) and with release dates, Lu et al. [14] proved that it is unary NP-hard and proposed a 2-approximation algorithm and a PTAS. Cao and Yang [15] also developed a PTAS.

As for the problem with the total completion time as part of the objective, Li and Feng [16] considered a single batching machine with unlimited capacity and the objective being the sum of the total completion time and the total rejection cost. They showed that the problem can be solved in $O(n^3 \log n)$ time. Zhang et al. [17] considered the same problem and proposed an optimal algorithm with running time $O(n^3)$. They also considered the problem of finding a feasible schedule with the total completion time and the total rejection cost not exceeding certain bounds and proved that this problem is binary NP-complete. They developed a pseudo-polynomial time algorithm as well as a Fully Polynomial-Time Approximation Scheme (FPTAS) for its optimization version.

There are a few more results dealing with batch scheduling problems with rejection in slightly different contexts. Miao et al. [18] considered unrelated parallel batching machines with limited capacities, with rejection, and with the processing times being only dependent on the machine. For the case in which the objective is the minimization of the total weighted completion time plus the total rejection cost, they designed a pseudo-polynomial time algorithm with a running time of $O(mn^3 p_{\max} \sum w_j)$ where m is the number of machines and $p_{\max}$ is the maximum processing time of the n jobs. For the objective of minimizing the makespan plus the total rejection cost, they designed a pseudo-polynomial time algorithm with a running time of $O(mn^2 \sum w_j)$. Shabtay [19] considered a single machine scheduling problem with serial batching and rejection and as objective the sum of the total completion time and the total rejection cost.

In this paper we use the following notation throughout. Let σ denote a feasible schedule. Let $\mathcal{A}(\sigma)$ and $\mathcal{R}(\sigma)$ denote the sets of accepted jobs and rejected jobs under schedule σ, respectively. Clearly,

$$\mathcal{J} = \mathcal{A}(\sigma) \cup \mathcal{R}(\sigma).$$

Let $RC(\sigma)$ denote the total rejection cost under schedule σ. The completion time of job j under schedule σ is denoted by $C_j(\sigma)$.

We will use the common three-field notation, $\alpha \mid \beta \mid \gamma$, to describe our problems. The α stands for the machine environment, the entries in the β field contain all the job and processing characteristics, and the γ represents the objective to be minimized [20]. For example, $1 \mid batch(b \geq n) \mid C_{\max}$ implies a single batching machine problem with the machine

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