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Binary linear programming models for robust broadcasting in communication networks

Ronald G. McGarvey [a,*], Brian Q. Rieksts [b], José A. Ventura [c,1], Namsu Ahn [d]

[a] Department of Industrial and Manufacturing Systems Engineering, Harry S. Truman School of Public Affairs, University of Missouri, E3437D Lafferre Hall, Columbia, MO 65211, United States

[b] Institute for Defense Analyses, Cost Analysis and Research Division, 4850 Mark Center Drive, Alexandria, VA 22311, United States

[c] Harold and Inge Marcus Department of Industrial and Manufacturing Engineering, The Pennsylvania State University, 310 Leonhard Building, University Park, PA 16802, United States

[d] Department of Industrial Management, Ulsan College, Ulsan, South Korea

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A B S T R A C T

Broadcasting is an information dissemination process in communication networks whereby a message, originated at any node of a network, is transmitted to all other nodes of the network. In c-broadcasting, each node having the message completes up to c transmissions to its neighbors over the communication lines in one time unit. In a k-fault tolerant c-broadcast network, the broadcasting process can be accomplished even if k communication lines fail. This paper presents innovative binary linear programming formulations to construct c-broadcast graphs, k-fault-tolerant c-broadcast graphs, and their time-relaxed versions. The proposed mathematical models are used to generate eight previously unknown minimum c-broadcast graphs, new upper bounds for eleven other instances of the c-broadcast problem, and over 30 minimum k-fault-tolerant c-broadcast graphs. The paper also provides a construction method to produce an upper bound for an infinite family of k-fault-tolerant c-broadcast graphs.

1. Introduction

Communication is a propagation process of information between a set of information producers and a set of information consumers through logical and physical media. Logical media refers to the protocol that is used, such as language, symbols, and numbers, while physical media represents the communication links, including cables, radio, and light. When there are more than two members involved in the propagation process, it is necessary to establish the scheme to link them satisfying certain goals. The linking scheme is the communication network. Thus, the topology of the network has a significant influence on its performance and operational cost. Examples of communication networks that need to be designed efficiently and economically are abundant in many application areas, such as computer networks, telecommunication networks, and interconnection networks [6].

Broadcasting can be considered as the most basic information dissemination process. It is the process of disseminating a message from a given node (originator) to all other nodes in a network. After receiving the message, a node may transmit

* Corresponding author. Tel.: +1 573 882 9564.
E-mail addresses: mcgarveyr@missouri.edu (R.G. McGarvey), brieksts@ida.org (B.Q. Rieksts), jav1@psu.edu (J.A. Ventura), namsu.ahn@gmail.com (N. Ahn).
1 Tel.: +1 814 865 3841; fax: +1 814 863 4745.

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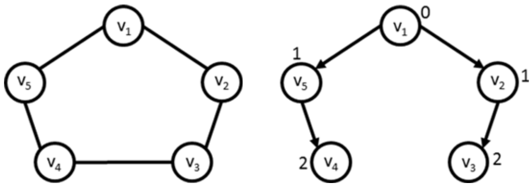

(a) 2-mbg with five nodes. (b) Broadcast protocol.

Fig. 1. 2-mbg with five nodes and its broadcast protocol.

the message to its adjacent nodes. In regular broadcasting [14], a node can transmit the message to only one adjacent node per time unit, where a time unit is defined as the transmission time of the message, but different pairs of nodes may communicate simultaneously. In c-broadcasting [21], which is a generalization of regular broadcasting, up to c adjacent nodes are permitted to receive the message from a single node per time unit.

A communication network can be modeled as a connected undirected graph $G(V, E)$, where V is the set of nodes and E the set of communication links (edges). For a given graph G and originator $v \in V$, the minimum time to complete c-broadcasting from v is defined as $b_c(v)$. Thus, the c-broadcast time of graph G across all potential originator nodes is $b_c(G) = \max\{b_c(v)|v \in V\}$. Note that, if p nodes have the message at the beginning of a time unit, at most $(c+1)p$ nodes can have the message at the end of the time unit. Thus, for an arbitrary graph G of order n, i.e., $n = |V|$, the minimum possible value for $b_c(G)$ is $\lceil \log_{c+1} n \rceil$. A graph with this property is called a c-broadcast graph (c-bg). The minimum number of edges of a c-bg of order n is given by the c-broadcast function $B_c(n)$. A c-bg of order n with $B_c(n)$ edges is said to be a minimum c-broadcast graph (c-mbg). The message transmission process originated at node v in c-broadcasting determines a spanning tree. A *c-broadcast protocol* (or *c-broadcast tree*) is a rooted spanning tree with n nodes in which the originator v is the root and all nodes are labeled by their receiving times, which are equal to or less than $b_c(G)$.

Fig. 1 shows a 2-mbg with five nodes and its broadcast protocol. Since all nodes are topologically equivalent, the 2-broadcast protocol in Fig. 1(b) is valid for any originator. The number next to each node indicates the time that the node receives the message.

In addition to the c-broadcast function, some related problems will be discussed over the course of this paper. One such problem is time-relaxed c-broadcasting. For this case, additional time units are allowed by restricting $b_{t,c}(G) = \lceil \log_{c+1} n \rceil + t$ instead of $b_c(G) = \lceil \log_{c+1} n \rceil$. The function $B_{t,c}(n)$ is the minimum number of edges of a time-relaxed c-broadcast graph ((t, c)-rbg) of order n with $b_c(G) \le \lceil \log_{c+1} n \rceil + t$.

Another problem of interest is the k-fault tolerant (k-ft) broadcast problem [38]. The literature in this area has focused on single-transmission broadcasting, i.e., $c = 1$. These graphs are structured such that the message reaches its destination even in the event of permanent failures in no more than k communication links. The objective is to construct sparse graphs with reliable transmission schemes. The protocols of a k-ft broadcast graph are predefined in such a way that if any k edges in the protocol fail, the message will still reach all of the non-originators in the graph. If there is a common edge in all paths to a given node in some protocol, this edge could fail, preventing the node from receiving the message. Thus, a k-ft protocol must contain $k+1$ independent (edge disjoint) paths to each non-originator in the network. The constraints of the regular broadcast problem also apply to the k-ft broadcast problem. During a particular time unit, a node may not send and receive messages from two different nodes simultaneously according to Liestman [38]. In a message transfer between adjacent nodes, it is possible for a node to both send and receive the message from a single node during the same time unit. In this case, the message becomes scrambled if both nodes have the message, but this is irrelevant since they already have the message. If there is a failure and one of the nodes does not have the message, a transmission is sent from the node with the message to the node without the message. Liestman [38] proves $\lceil \log_2 n \rceil + 1$ is the minimum number of time units required for completing a 1-ft broadcast protocol in a network of three or more nodes. For the general k-ft problem, there is not a known closed form expression for the minimum broadcast time for every instance, but several results provide the minimum broadcast time for instances satisfying certain properties. For example, Liestman [38] proves $\lceil \log_2 n \rceil + 1$ is the minimum number of time units required for completing a 1-ft broadcast protocol in a network of three or more nodes. A summary of these closed form results for the k-ft problem is presented in [1].

A broadcast network that permits k-ft protocols from each originator in minimum time is called a k-fault tolerant broadcast graph (k-ftbg). Although the literature focuses on $c = 1$, in this paper we extend these concepts to address general k-ft c-broadcast graphs ((c, k)-bg's). The function $B_{c,k}(n)$ is defined as the minimum number of edges of all (c, k)-bg's with n nodes. We note that $B_{c,k=0}(n) = B_c(n)$. A graph with n nodes and $B_{c,k}(n)$ edges is called a k-fault-tolerant minimum c-broadcast graphs ((c, k)-mbg's). Fig. 2(a) provides an example of a $(1, 1)$-mbg with four nodes. In Fig. 2(b), a 1-ft protocol is displayed with node v_1 being the originator. This protocol is separated into two parts to show two independent paths to each node. During the first two time units, each node receives the message if failures do not occur, but it is necessary to continue to broadcast to provide two independent paths to each non-originator. During the last time unit (time unit three), a bi-directional exchange takes place between nodes v_2 and v_3 (denoted by the dashed lines), and a message is sent from node v_1 to node v_4. Since the protocol contains two independent paths to each non-originator, is completed in $\lceil \log_2 4 \rceil + 1 = 3$