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Directed elimination games

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ABSTRACT

While tools from structural graph theory such as tree- or path-width have proved to be highly successful in coping with computational intractability on undirected graphs, corresponding width measures for directed graphs have not yet fulfilled their promise for broad algorithmic applications on directed graphs. One reason for this is that in most existing digraph width measures the class of acyclic digraphs has small width which immediately implies hardness of problems such as computing directed dominating sets.

Fernau and Meister (2014) introduce the concept of elimination width and a corresponding graph searching game which overcomes this problem with acyclic digraphs. In their paper, the focus was on structural characterizations of classes of digraphs of bounded elimination width.

In this paper we study elimination width from an algorithmic and graph searching perspective. We analyse variations of the elimination width game and show that this leads to width measures on which the directed dominating set problem and some variants of it become tractable.

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1. Introduction

Structural graph theory has proved to be a powerful tool for coping with computational intractability. It provides a wealth of concepts and results that can be used to design efficient algorithms for hard computational problems on specific classes of graphs that occur naturally in applications. For instance, it has been shown that many problems which in general are NP-complete become tractable on planar graphs or graphs of bounded genus. Here, tractable can mean solvable in polynomial time or fixed-parameter tractable, the analogous concept of efficient solvability in *parameterized complexity* (see, e.g., [12]).

Of particular importance in this context are structural properties such as tree-width or path-width introduced by Robertson and Seymour as part of their celebrated graph minor project [27]. See, e.g., [11] for an introduction to these concepts. Most problems that are tractable on trees also remain tractable on classes of graphs of bounded tree-width and a huge number of papers are devoted to developing efficient algorithms for a wide range of problems on such classes of graphs. See, e.g., [5,6] and references therein.

However, in many applications in computer science, directed graphs are the more natural model. Unfortunately, so far research in structural graph theory has almost exclusively focused on undirected graphs and no structure theory for directed graphs has been developed that would provide for a similar set of tools and concepts to deal with hard computational problems on digraphs.

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Reed [26] and Johnson et al. [20] initiated the development of a decomposition theory for directed graphs with the aim of defining an analogue of the concept of undirected tree-width for directed graphs. Following their definition of a *directed tree-width*, several alternative notions have been introduced, for instance in [28,3,4,19]. For each of these decompositions and associated width measure it was shown that several problems become tractable if the width of digraphs with respect to these measures is bounded by a fixed constant. However, most examples of problems becoming tractable are either linkage problems, i.e. problems asking for the existence of certain pairwise disjoint paths in the digraph, or certain combinatorial games arising in verification. The only exception is *bi-rank-width*, which was introduced by Kanté et al. [21] for a very different purpose than the other digraph width measures as part of the ongoing quest to find suitable structural parameters generalizing the concept of clique-width to more general structures than undirected graphs. On classes of digraphs of bounded bi-rank-width all problems definable in *monadic second-order logic* (MSO) become tractable. However, in the context of algorithmic applications to digraphs, monadic second-order logic is too powerful, as it allows to specify on a directed graph properties of its underlying undirected graph. Bi-rank-width is therefore a special case in digraph width measures as bounded bi-rank-width implies bounded clique-width of the underlying undirected graphs whereas the other width measures were introduced specifically so that classes of bounded width do not also have bounded tree- or clique-width.

Following these initial proposals for directed analogues of tree-width, several papers investigated how broad the algorithmic theory of classes of digraphs of bounded width with respect to these measures is. Unfortunately, for many interesting problems other than those mentioned before, strong intractability results were obtained showing that the algorithmic applicability of the existing directed width measures is very limited. See, e.g., [24,22,9,16,17] and references therein.

In nearly all existing digraph width measures the class of acyclic digraphs (DAGs) has very small width. The negative results reported above show very clearly that exactly this assumption, that DAGs are "simple", is a major obstacle for algorithmic applications of directed width measures, as standard problems such as the directed dominating set problem are NP-complete and fixed-parameter intractable already on very simple acyclic digraphs (see Section 7 for details). Hence, if we aim for a width measure which can be used in the analysis of problems such as directed dominating sets, then it must necessarily split the class of DAGs into simple and hard instances.

In [25], a concept called *nowhere crownful* classes of digraphs was introduced which achieves this goal. Nowhere crownful classes are very general, in particular they include all classes of digraphs whose underlying undirected graphs exclude a fixed minor or are nowhere dense. And yet problems such as the directed dominating set problem and others become fixed-parameter tractable. However, this generality comes at a price as it is easily seen that disjoint paths problems remain hard on nowhere crownful classes of digraphs. Hence, more research and different ideas are needed to find parameters on which more and different problems become tractable.

Elimination width. An elegant and promising proposal for such a parameter was given by Fernau and Meister in [13]. Like many other digraph width parameters, in particular directed tree-width [20], DAG-width [4] and Kelly-width [19], it is based on *graph searching games*. In a graph searching game, a number of cops try to catch a robber hiding on the vertices of a graph or digraph. That is, in every round of the play, the robber occupies a vertex and so does each of the cops. The game then proceeds as follows. In every round the cop player can lift some cops and place them on arbitrary new vertices. While the cops are in transit from their old to the new positions, the robber can also change his position. The different variants of the game are determined by the rules the robber has to follow when moving to a new position, but in all graph searching games, the robber is not allowed to move through a cop which has not been moved. The cops win if they can place a cop on the position occupied by the robber without the robber being able to escape. Otherwise, i.e. if the robber can escape forever, then he wins. See Section 3 for details and see [7,15,23] for surveys and books of the types of games relevant for this paper.

Graph searching games yield a natural graph invariant, namely the minimal number of cops needed to catch a robber on a given graph. Naturally, different versions of the game yield different graph invariants and there are games corresponding exactly to the tree- or the path-width of a graph. For directed graphs, many width measures can also be defined by graph searching games, namely the *strongly connected visible robber game* for directed tree-width, *monotone directed visible robber game* for DAG-width, the *inert invisible robber game* for Kelly-width and the *agile, invisible robber game* for directed path-width. In all of these games, the robber can only move along directed paths and this immediately implies that on DAGs a very small number of cops suffices to capture the robber: all they have to do is to force the robber along a directed path until he reaches a sink, i.e. a vertex with no successor.

The novel idea proposed in [13] is to consider games, called *directed elimination games*, where the robber can run along a directed path or *against* a directed path, i.e. choose a new position reachable from his current position by a directed path or from which there is a directed path to his current position. In this way they obtain games where the number of cops needed to catch a robber on the class of DAGs is unbounded. The main purpose of their work in [13] is to study the structure of graphs of bounded elimination width, i.e. on which a bounded number of cops suffices to capture the robber in the elimination game. In particular, it turns out that such classes can equivalently be defined by perfect elimination orderings and other structural characterizations, i.e. form a robust class of directed graphs.

Our contributions. The purpose of this paper is to further explore elimination games and in particular study their applications for algorithmic digraph problems. The variant of the game studied in [13] is what is called the *inert invisible robber game* in the graph searching literature. As explained above, on directed graphs, different variants of graph searching games yield very different width measures with completely different properties. In this paper we therefore first study the other standard variants of graph searching games equipped with the elimination width idea of robber movement. Apart from

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