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Characterizations of cographs as intersection graphs of paths on a grid

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ABSTRACT

A cograph is a graph which does not contain any induced path on four vertices. In this paper, we characterize those cographs that are intersection graphs of paths on a grid in the following two cases: (i) the paths on the grid all have at most one bend and the intersections concern edges ($\rightarrow B_1$-EPG); (ii) the paths on the grid are not bended and the intersections concern vertices ($\rightarrow B_0$-VPG).

In both cases, we give a characterization by a family of forbidden induced subgraphs. We further present linear-time algorithms to recognize B_1-EPG cographs and B_0-VPG cographs using their cotree.

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1. Introduction

Edge intersection graphs of paths on a grid (or EPG graphs) are graphs whose vertices can be represented as paths on a rectangular grid such that two vertices are adjacent if and only if the corresponding paths share at least one edge of the grid. We may assume that the grid is $\mathbb{Z}^2$ or a sufficiently large subset of it. The EPG graphs were first introduced in [16] and have been studied by several authors (see for instance [2,3,5,22,23]). Every graph G is an EPG graph [16], so motivated by the study of these graphs with constraints from circuit layout problems, Golumbic, Lipshteyn and Stern introduced subclasses of EPG graphs based on restricting the number of bends permitted for each path. Specifically, for a fixed $k \geq 0$, the paths on the grid that represent the vertices of a graph are allowed to have at most k bends, i.e., at most k grid point turns, and the subclass of graphs that admit such a representation is denoted by B_k-EPG. Notice that B_0-EPG graphs are equivalent to interval graphs.

In [3], the authors show that for any k, only a small fraction of all labeled graphs on n vertices are B_k-EPG. Some results of [3] were also proved in [5]. In addition, the authors of [5] consider different classes of graphs and show, in particular, that every planar graph is a B_5-EPG graph. This result was later improved in [23], where the authors show that every planar graph is a B_4-EPG graph. It is still open if $k = 4$ is best possible. So far it is only known that there are planar graphs that are B_3-EPG graphs and not B_2-EPG graphs. The authors in [23] also show that all outerplanar graphs are B_2-EPG graphs thus proving a conjecture of [5].

For the case of B_1-EPG graphs, Golumbic, Lipshteyn and Stern [16] showed that every tree is a B_1-EPG graph, and Asinowski and Ries [2] showed that every B_1-EPG graph on n vertices contains either a clique or a stable set of size at least $n^{1/3}$. The problem of recognizing B_1-EPG graphs was shown to be NP-complete by Heldt, Knauer and Ueckerdt in [22]. It is

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therefore interesting to see which subfamilies of B_1-EPG graphs have special properties and which can be efficiently recognized. Asinowski and Ries [2] give a characterization of the B_1-EPG graphs among some subclasses of chordal graphs, namely, chordal bull-free graphs, chordal claw-free graphs, chordal diamond-free graphs, and special cases of split graphs. It follows from [16,3] that a complete bipartite graph $K_{m,n}$ ($m \leq n$) is B_1-EPG if and only if $m \leq 2$ and $n \leq 4$. Since complete bipartite graphs are a special case of cographs, it is natural to ask for a characterization of B_1-EPG cographs. In [12], it is proven that cographs are well quasi ordered with respect to the induced subgraph relation. Therefore, any subfamily of the class of cographs can be characterized by a finite set of forbidden minimal induced subgraphs and recognized in polynomial time. However, it is only proven that such obstruction set exists. In Section 4 of this paper, we provide such a characterization for B_1-EPG by giving a complete family of minimal forbidden induced subgraphs. Later, in Section 6, we present an efficient linear-time algorithm to recognize this subfamily using their cotrees.

Instead of considering edge intersection graphs of paths on a grid, one may be interested in vertex intersection graphs of paths on a grid (or VPG graphs). The VPG graphs are graphs whose vertices correspond to paths on a rectangular grid such that two vertices are adjacent if and only if the corresponding paths share at least one grid point. These graphs were first introduced in [1] and have also been studied by several authors (see for instance [7,8,17]). In [1], the authors show that VPG graphs are exactly string graphs, i.e., intersection graphs of arbitrary curves in the plane. As in the case of EPG graphs, one may restrict the number of bends for each path. Hence, for a fixed $k \geq 0$, the paths on the grid that represent the vertices of a graph are allowed to have at most k bends, i.e., at most k grid point turns, and the subclass of graphs that admit such a representation is denoted by B_k-VPG. In [1], the authors notice that B_0-VPG graphs are equivalent to the so called 2-DIR graphs, whose recognition complexity is NP-complete [24].

A hierarchy of VPG graphs, relating them to other known families of graphs, is presented in [1], where they show for instance that planar graphs are B_3-VPG graphs. This result was recently improved in [8] where it was shown that planar graphs are B_2-VPG graphs. It remains open if $k = 2$ is best possible for planar graphs. In [17], the authors characterize split graphs that are B_0-VPG graphs by giving a family of minimal forbidden induced subgraphs. Furthermore, they characterize chordal claw-free B_0-VPG graphs as well as chordal bull-free B_0-VPG graphs. It is easy to see that all permutation graphs are B_1-VPG by labeling the x and y axes with the two permutations and connecting each pair of numbers with a single bend path. It thus follows that cographs (a subfamily of permutation graphs) are B_1-VPG. So it is natural to ask which cographs are B_0-VPG. In Section 5 of this paper, we characterize the B_0-VPG cographs as those which contain no induced 4-wheel, and present an efficient linear-time recognition algorithm using the cotree of the graph in Section 6.

We start with some preliminaries in Section 2, and in Section 3 we present some useful basic properties of the neighborhoods of C_4's in cographs which will be useful in our proofs characterizing B_1-EPG cographs. In Sections 4 and 5 we present characterizations for the classes of B_1-EPG cographs and B_0-VPG cographs, respectively. Linear time recognition algorithms for both of these classes are given in Section 6. Finally, we conclude with some open questions in Section 7. For graph theoretical terms that are not defined here, we refer the reader to [14,25].

2. Preliminaries

2.1. General graph definitions and notation

All graphs in this paper are connected, finite and simple. A *clique* is a set of pairwise adjacent vertices and a *stable set* is a set of pairwise nonadjacent vertices. The size of a maximum stable set in G is called the *stability number* of G and is denoted by $\alpha(G)$. A set $U \subseteq V$ is called *dominating* if for every vertex $v \in V \setminus U$ there exists $u \in U$ such that $uv \in E$. For disjoint sets $A, B \subseteq V$, we say that A is *complete* to B if every vertex in A is adjacent to every vertex in B, and that A is *anticomplete* to B if every vertex in A is nonadjacent to every vertex in B. The complement of a graph G will be denoted by $\overline{G}$. As usual, C_k, $k \geq 3$, denotes an induced cycle on k vertices. A vertex v which is adjacent to all the vertices of a C_k is called a *center*, and we call the graph induced by $V(C_k) \cup \{v\}$ a *k-wheel* denoting it by W_k (although it has $k+1$ vertices). Finally, P_k, $k \geq 0$, denotes an *induced path* on k vertices, K_p, $p \geq 0$, denotes a *clique* on p vertices, mK_p, $m, p \geq 0$, denotes m disjoint copies of mK_p, and $K_{p,q}$ denotes the *complete bipartite graph* with p vertices in one set of the bipartition and q vertices in the other set of the bipartition. More generally, $K_{m_1,\ldots,m_t}$ is the *complete multipartite graph* with part-sizes $m_1, \ldots, m_t$.

Let $G = (V, E)$ be a graph. For a vertex $v \in V$, we let $\mathcal{N}_G(v)$ denote the set of vertices in G that are adjacent to v, i.e., the neighbors of v. $\mathcal{N}_G(v)$ is called the *neighborhood* of vertex v. We will write $\mathcal{N}_G[v] = N_G(v) \cup \{v\}$, and call $\mathcal{N}_G[v]$ the *closed neighborhood* of vertex v. Whenever it is clear from the context what G is, we will drop the subscripts and write $\mathcal{N}(v) = \mathcal{N}_G(v)$ and $\mathcal{N}[v] = \mathcal{N}_G[v]$. A vertex v is called a *true twin* of some vertex u if $\mathcal{N}[v] = \mathcal{N}[u]$. We will denote by $G[X]$ the subgraph induced by $X \subseteq V$. We write $G - v$ for the subgraph obtained by deleting vertex v and all the edges incident to v. Similarly, for $A \subseteq V$, we denote by $G - A$ the subgraph of G obtained by deleting the set A and all the edges incident to some vertex in A, i.e., $G - A = G[V \setminus A]$.

We will denote by G_R the *reduced graph* of G, that is, the graph obtained from G by deleting for each set U of true twins all but one $u \in U$. Thus, G_R does not contain any pair of adjacent vertices which have exactly the same closed neighborhood. The next lemma immediately follows from the definition of the reduced graph G_R.

Lemma 1. *Let G be a graph and let G_R be its reduced graph. Then any connected component of G_R isomorphic to a clique is an isolated vertex.*

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