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Interval incidence coloring of bipartite graphs[☆]

Robert Janczewski [*], Anna Małafiejska, Michał Małafiejski

Department of Algorithms and Systems Modelling, Gdańsk University of Technology, Narutowicza 11/12, Gdańsk, Poland

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ABSTRACT

In this paper[1] we study the problem of interval incidence coloring of bipartite graphs. We show the upper bound for interval incidence coloring number (χ_{ii}) for bipartite graphs $\chi_{ii} \leq 2\Delta$, and we prove that $\chi_{ii} = 2\Delta$ holds for regular bipartite graphs. We solve this problem for subcubic bipartite graphs, i.e. we fully characterize the subcubic graphs that admit 4, 5 or 6 coloring, and we construct a linear time exact algorithm for subcubic bipartite graphs. We also study the problem for bipartite graphs with $\Delta = 4$ and we show that 5-coloring is easy and 6-coloring is hard ($\mathcal{NP}$-complete). Moreover, we construct an $O(n\Delta^{3.5} \log \Delta)$ time optimal algorithm for trees.

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1. Introduction

1.1. Problem definition

In the following we consider connected simple nonempty graphs only, and we use standard notations of the graph theory. For a given simple graph $G = (V, E)$, we define an *incidence* as a pair (v, e), where vertex $v \in V$ is one of the endpoints of edge $e \in E$. The set of all incidences of G will be denoted[2] by I, thus $I := \{(v, e): v \in V \wedge e \in E \wedge v \in e\}$. We say that two incidences (v, e) and (w, f) are *adjacent* if and only if one of the following holds: (1) $v = w$ and $e \neq f$; (2) $e = f$ and $v \neq w$; (3) $e = \{v, w\}, f = \{w, u\}$ and $v \neq u$.

By an *incidence coloring* of G we mean a function $c: I \to \mathbb{N}$ such that $c(v, e) \neq c(w, f)$ for any adjacent incidences (v, e) and (w, f). The *incidence coloring number* of G, denoted by χ_i, is the smallest number of colors in an incidence coloring of G.

A finite nonempty set $A \subseteq \mathbb{N}$ is an *interval* if and only if it contains all integers between $\min A$ and $\max A$. For a given incidence coloring c of graph G and $v \in V$ let $A_c(v) := \{c(v, e): v \in e \wedge e \in E\}$. By an *interval incidence coloring* of graph G we mean an incidence coloring c of G such that for each vertex $v \in V$ the set $A_c(v)$ is an interval. By an *interval incidence k-coloring* we mean a coloring using all colors from the set $\{1, \ldots, k\}$. The *interval incidence coloring number* of G, denoted by $\chi_{ii}(G)$, is the smallest number of colors in an interval incidence coloring of G. An interval incidence coloring of G using χ_{ii} colors is said to be *minimal*. We say that $v \in V$ is *minimal* if $\min A_c(v) = \min c(I)$, and we say that $v \in V$ is *maximal* if $\max A_c(v) = \max c(I)$.

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* Corresponding author. Tel.: +48 58 347 10 64.
E-mail addresses: skalar@eti.pg.gda.pl (R. Janczewski), aniam@kaims.pl (A. Małafiejska), mima@kaims.pl (M. Małafiejski).

1 The extended abstract of this paper appeared on PPAM 2009, LNCS 6067 (2010) 11–20.

2 To simplify notation, we write I instead of $I(G)$ whenever G is clear from the context. The same rule applies to other parameters of G appearing in the paper.

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1.2. Background, previous results and our contribution

In this paper we consider a restriction of the incidence graph coloring problem, in which the colors of incidences at a vertex form an interval. The considerations in this paper are motivated by the multicasting communication in a multifiber WDM (*wavelength-division multiplexing*) all-optical star network [2,4,5].

Alon et al. [1] defined a problem of partitioning a graph into a minimal number of star forests as a model of the possible fastest exchanging messages in a network with the assumption of the ability of sending messages to all neighbors at the same time and blocking receiving more than one message (a dual model is also possible). Brualdi and Massey [6] formulated a model of *incidence graph coloring* with references to some other models of graph coloring, such as strong edge and vertex coloring of graphs. Guiduli [14] observed that the problem of incidence graph coloring is a special case of the problem of partitioning a symmetric digraph into directed star forests. See [7,9,8,19,20] for more information about the incidence coloring of graphs.

The *interval edge coloring* of graphs was proposed by Asratian and Kamalian [3] who analyzed the complexity and the basic properties of interval edge coloring, defined also as consecutive coloring. A detailed review of the interval edge coloring problem is described in [11,13]. This problem has applications in open-shop scheduling, especially in dedicated systems without breaks [12].

In [16] the authors introduced the concept of *interval incidence coloring* modeling message passing in networks, and in [17] the authors studied the applications in a model of one-multicast transmission per node in multifiber WDM networks. In [18] the authors proved some lower and upper bounds for the *interval incidence coloring number* (χ_{ii}), and determined the exact values of χ_{ii} for selected classes of graphs: paths, cycles, stars, wheels, fans, necklaces, complete graphs and complete k-partite graphs. In [18] the authors also studied the complexity of the interval incidence coloring problem for subcubic graphs for which they showed that the problem whether $\chi_{ii} \le 4$ is easy, and $\chi_{ii} \le 5$ is $\mathcal{NP}$-complete.

In this paper, we study the problem of interval incidence coloring for bipartite graphs. In Section 2 we obtain an upper bound for the interval incidence coloring number of bipartite graphs, namely $\chi_{ii} \le 2\Delta$, and we prove that $\chi_{ii} = 2\Delta$ holds for regular bipartite graphs. In Section 3 we construct a linear time exact algorithm for subcubic bipartite graphs. In Section 4 we study the problem for bipartite graphs with $\Delta = 4$ and we show that 5-coloring is easy and 6-coloring is hard ($\mathcal{NP}$-complete). In Section 5 we construct a polynomial time exact algorithm for trees. Moreover, we fully characterize all bipartite graphs that admit 4-colorings (Section 3.1) and 5-colorings (Sections 3.2 and 4.1).

2. Bounds on χ_{ii} for bipartite graphs

In this section, we construct some lower and upper bounds on interval incidence coloring number for bipartite graphs. Observe that $\chi_i \le \chi_{ii}$ and hence any lower bound for χ_i is a lower bound for χ_{ii}.

Theorem 1. *For any nonempty bipartite graph G we have*

$$\Delta + 1 \le \chi_i \le \chi_{ii} \le 2\Delta.$$

Proof. It is easy to prove that the first two inequalities hold for all nonempty graphs and therefore we omit this part of the proof.

To prove the right-hand side inequality, we divide the vertex set into 2 independent sets denoted by V_1 and V_2. We create a coloring c from I to $\mathbb{N}$ in the following way: if $v \in V_1$, then we assign colors to incidences at vertex v (i.e. of form (v, e)) in such a way that $A_c(v) = \{1, \dots, \deg v\}$, and if $v \in V_2$, then we assign $A_c(v) = \{\Delta + 1, \dots, \Delta + \deg v\}$. Hence we have $A_c(v) \cap A_c(w) = \emptyset$ for any $v \in V_1$ and $w \in V_2$, thus c is an interval incidence 2Δ-coloring of G. □

Let G be any regular bipartite graph of degree Δ. It is easy to observe, that in any interval incidence χ_{ii}-coloring c of G, there is at least one vertex v that is minimal, i.e. $\min A_c(v) = 1$. Moreover, there is a vertex u adjacent to v such that $c(v, \{v, u\}) = \Delta$, hence $\min A_c(u) > \Delta$, and by Theorem 1 we have

Theorem 2. $\chi_{ii}(G) = 2\Delta$ *for any regular bipartite graph G.* □

3. Polynomial time algorithm for subcubic bipartite graphs

In this section, we focus on the interval incidence coloring problem for subcubic bipartite graphs. Observe, that $\chi_{ii}(P_2) = 2$ and $\chi_{ii}(P_3) = \chi_{ii}(P_4) = 3$, where P_n is a n-vertex path. If G is a path with at least 5 vertices or a cycle, then it is easy to observe that $\chi_{ii}(G) = 4$. In the following, let G be a bipartite graph with $\Delta(G) = 3$. By Theorem 1 the interval incidence chromatic number $\chi_{ii}(G)$ is between 4 and 6. We construct an efficient algorithm for coloring such graphs with minimum number of colors (i.e. using 4, 5 or 6 colors).

3.1. Interval incidence 4-coloring of bipartite graphs with $\Delta = 3$

Lemma 1. *If* $\chi_{ii}(G) = 4$ *then*

(i) *each vertex* v *of degree* 3 *has at most one neighbor of degree* 3,

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