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Communication

Diagnosis of constant faults in iteration-free circuits over monotone basis

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ABSTRACT

We show that for each iteration-free combinatorial circuit S over a basis B containing only monotone Boolean functions with at most five variables, there exists a decision tree for diagnosis of constant faults on inputs of gates with depth at most $7L(S)$ where $L(S)$ is the number of gates in S.

1. Introduction

Diagnosis of faults is a well-established area of research in computer science with a long enough history: the early results were published in the fifties of the last century [3,5,10,14]. This paper is closely connected with the study of decision trees for diagnosis of constant faults on inputs of gates in iteration-free (tree-like) combinatorial circuits over arbitrary finite basis B of Boolean functions.

A Boolean function is called linear if it can be represented in the form $\delta_1 x_1 + \cdots + \delta_n x_n + \delta_{n+1} (\mathrm{mod}\ 2)$ where $\delta_1, \ldots, \delta_{n+1} \in \{0, 1\}$. A Boolean function is called unate if it can be represented in the form $f(x_1^{\sigma_1}, \ldots, x_n^{\sigma_n})$ where $f(x_1, \ldots, x_n)$ is a monotone Boolean function, $\sigma_1, \ldots, \sigma_n \in \{0, 1\}$, and $x^1 = x$ and $x^0 = \neg x$.

From the results obtained in [11,12], which generalize essentially the results presented in [7,9], it follows that if B contains only linear or only unate Boolean functions then the minimum depth of decision trees in the worst case grows at most linearly depending on the number of gates in the circuits, and it grows exponentially if B contains a non-linear function and a function which is not unate.

In this paper, for simplicity, we consider only monotone Boolean functions. Similar results can be obtained for unate Boolean functions.

The results obtained in [11,12,8] imply that, for each iteration-free combinatorial circuit S over a basis B containing only monotone Boolean functions with at most n variables, there exists a decision tree for diagnosis of constant faults on inputs of gates with depth at most $\varphi(n)L(S)$, where $\varphi(n) = \binom{n}{\lfloor n/2 \rfloor} + \binom{n}{\lfloor n/2 \rfloor + 1}$ and $L(S)$ is the number of gates in S. We decrease the coefficient from $\varphi(n)$ to $n+1$ for $n = 1, 2, 3, 4$ and to $n+2$ for $n = 5$ (note that $\varphi(n) = n+1$ for $n = 1, 2$). The obtained bounds are sharp.

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Table 1
Number $M(n)$ of monotone Boolean functions with n, $1 \leq n \leq 5$, variables.

n	1	2	3	4	5
$M(n)$	3	6	20	168	7581

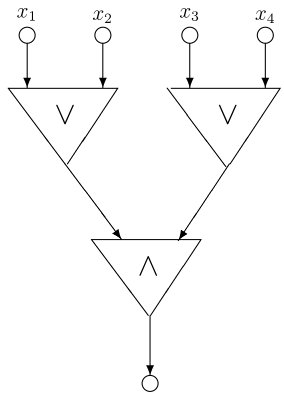

Fig. 1. Iteration-free circuit with 4 inputs, 3 gates and 6 inputs of gates over the basis $\{x \wedge y, x \vee y\}$.

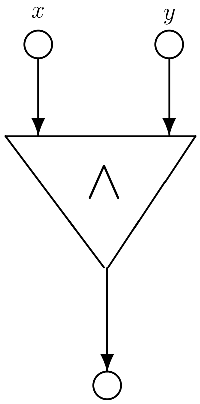

Fig. 2. Iteration-free circuit $S_\wedge$ with one gate.

2. Basic notions

A monotone Boolean function is a Boolean function $f(x_1, \ldots, x_n)$ satisfying the following condition: for any $(a_1, \ldots, a_n), (b_1, \ldots, b_n) \in \{0, 1\}^n$ if $a_1 \leq b_1, \ldots, a_n \leq b_n$ then $f(a_1, \ldots, a_n) \leq f(b_1, \ldots, b_n)$. We will consider only monotone Boolean functions with at least one variable. The number $M(n)$ of monotone Boolean functions with n, $1 \leq n \leq 5$, variables can be found in Table 1 (see [4]).

A finite set of monotone Boolean functions B will be called a monotone basis. An iteration-free circuit over the basis B is a tree-like combinatorial circuit in which inputs are labeled with pairwise different variables and gates are labeled with functions from B. Figs. 1 and 2 show examples of iteration-free circuits.

Let S be a circuit over B with m circuit inputs and k inputs of gates. We consider constant (0 and 1) faults on the inputs of the gates of S, i.e. we assume that some of the inputs of gates are faulty and have fixed constant values (0 or 1), regardless of the values supplied to the inputs. There are 3^k possible k-tuples of faults for a circuit with k inputs of gates (since for each input we have a normal state and two different faults). The problem of diagnosis of S is to recognize the function implemented by the circuit S which has one of the 3^k possible k-tuples of faults.

We solve this problem using decision trees. At each step, a decision tree gives a tuple from $\{0, 1\}^m$ on the inputs of S and observes the output of S. The depth of the decision tree is the maximum number of steps made by this tree. We denote by $h(S)$ the minimum depth of a decision tree for diagnosis of S.

Fig. 3 shows a decision tree of depth 3 for diagnosis of the one-gate circuit $S_\wedge$ depicted in Fig. 2. First, the decision tree asks about the output of faulty circuit $S_\wedge$ for the input tuple (0, 1). Let the output be equal to 0. Then the decision tree asks