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Note

Binary linear programming solutions and non-approximability for control problems in voting systems

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ABSTRACT

In this paper we consider constructive control by adding and deleting candidates in Copeland and Llull voting systems from a theoretical and an experimental point of view.

We show how to characterize the optimization versions of these four control problems as special digraph problems and binary linear programming formulations of linear size. Our digraph characterizations allow us to prove the hardness of approximations with absolute performance guarantee for optimal constructive control by deleting candidates in Copeland and by adding candidates in Llull voting schemes and the nonexistence of efficient approximation schemes for optimal constructive control by adding and deleting candidates in Copeland and Llull voting schemes. Our experimental study of running times using LP solvers shows that for a lot of practical instances even the hard control problems can be solved very efficiently.

1. Introduction

An *election* (C, V) consists of a finite set C of candidates and of a finite multiset V of votes. A *voting system* is a rule that determines the winners of a given election. In this paper we consider the voting systems, Copeland and Llull [13].

In order to change the outcome of an election several different approaches have been defined. We consider the model of control, where the task of the so-called chair is to make a favorite candidate win the election (constructive control) or to prevent a distinguished candidate from winning the election (destructive control) by adding or deleting a *bounded* number of candidates. For Copeland and Llull the time complexity of these control problems has been determined in [13].

In this paper we consider control problems $\mathcal{E}$-$\mathfrak{CT}$ defined by a voting system $\mathcal{E} \in \{\text{Copeland}, \text{Llull}\}$ and some of the four control types $\mathfrak{CT}$ obtained by constructive control or destructive control and adding or deleting candidates. Each of these control problems $\mathcal{E}$-$\mathfrak{CT}$ is transformed to an optimization version $\mathcal{E}$-O-$\mathfrak{CT}$ in which the task of the chair is to make a favorite candidate win the election (constructive control) or to prevent a distinguished candidate from winning the election (destructive control) by adding or deleting a *minimum* number of candidates. The time complexity of optimal constructive control by adding and deleting candidates in Copeland and Llull voting schemes is known to be hard. This motivates to give useful characterizations of the hard optimization problems as well as easy algorithms for their solutions. Both will be done using linear programming formulations, which is a very powerful tool with a history of more than 50 years [20]. Therefore in this paper we show how to transform all these four optimization control problems $\mathcal{E}$-O-$\mathfrak{CT}$ into equivalent digraph problems. The digraph problems are transformed into equivalent binary linear programs of linear size, i.e. using a linear number of variables and constraints with respect to the number of candidates.

While in [24] it has been shown that manipulation in Borda has an approximation algorithm with an absolute performance guarantee, we prove the nonexistence of approximation algorithms with absolute performance guarantees for

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Table 1
Results on control complexity of Copeland and Llull. R means resistance and V means vulnerability to a particular control type. The results are due to [13].

Control by	Constructive control (CC)		Destructive control (DC)	
	Deleting candidates (DC)	Adding candidates (AC)	Deleting candidates (DC)	Adding candidates (AC)
Copeland	R	R	V	V
Llull	R	R	V	V

optimal constructive control by deleting candidates in Copeland and by adding candidates in Llull voting schemes, unless P = NP. The known hardness of the standard parameterization of the corresponding decision problems is used to show the nonexistence of efficient approximation schemes. We have implemented our LP solutions using Matlab LP solvers. Our experimental results show that for instances up to 1000 candidates the control problems can be solved very efficiently. In particular optimal constructive control by adding candidates in Copeland and Llull voting can be done much faster than control by deleting candidates in Copeland and Llull voting.

2. Preliminaries

2.1. Elections and control problems

An *election* is a pair (C, V), where $C = \{c_1, \ldots, c_m\}$ is a finite set of candidates and $V = \{v_1, \ldots, v_n\}$ is a finite multiset of votes, where each voter expresses his or her preferences over the candidate set C. In this paper a vote is an ordered list, which is a permutation of all candidates without ties. A *voting system* is a rule that determines the winners of a given election. We use the unique-winner model, i.e. we define a candidate $c \in C$ as a winner of an election (C, V), if c is the only winner of the election (i.e., there are no other candidates tying for the first place).

In this paper we consider the voting systems Copeland and Llull, which are defined as follows. Let $\alpha \in [0, 1]$. In *Copeland*$^\alpha$ *voting* each voter has to specify a tie-free linear ordering of all candidates. Every pair of candidates (c_1, c_2) is compared within a head-to-head contest. Candidate c_1 wins against candidate c_2 in a head-to-head contest if it is better-positioned in more than half of the votes. In this case, the winner c_1 gets one point and the loser c_2 gets zero points. If the candidates are tied, they both get α points. The winner is the candidate with the highest score. Following the notations in [8], Copeland^0 is denoted as *Copeland* and Copeland^1 is denoted as *Llull*.

The outcome of an election can be affected in several ways such as with manipulation, bribery, or control. In *manipulation* [3,11] coalitions of voters cast their votes insincerely in order to reach their goal. In *bribery* [12,13] an external agent is allowed to change some voters' votes in order to reach his or her goal. In *control* [4,18,13,19] an external agent – usually called the chair – can change the structure of the election (for example by adding or deleting either candidates or voters) in order to change the outcome of the election. In this paper we are only considering control. The chair can have two different intentions. First, the chair's goal could be to make his or her favorite candidate win the election (*constructive control*) [4], second, the chair's goal could be to bar a distinguished candidate from winning the election (*destructive control*) [18]. For the voting systems $\mathcal{E} \in$ {Copeland, Llull} we consider the following four control problems.

Name $\mathcal{E}$-Constructive Control by Adding Candidates ($\mathcal{E}$-CC-AC)
Instance An election $(C \cup D, V)$, where C is a set of candidates and D is a set of spoiler candidates with $C \cap D = \emptyset$, a distinguished candidate $c \in C$, and a positive integer k.
Question Is there a subset $D' \subseteq D$ of size at most k, such that c is the unique $\mathcal{E}$ winner of the election $(C \cup D', V)$?

Control by adding candidates models candidate recruitment. For practical examples see [5,4,18,13]. $\mathcal{E}$-Destructive Control by Adding Candidates ($\mathcal{E}$-DC-AC) is defined analogously with the difference that we ask whether it is possible to keep candidate c from being the unique $\mathcal{E}$ winner of the election $(C \cup D', V)$.

Next we will define control by deleting candidates, which models actions where candidates are being forced out of race.

Name $\mathcal{E}$-Constructive Control by Deleting Candidates ($\mathcal{E}$-CC-DC)
Instance An election (C, V), a distinguished candidate $c \in C$, and a positive integer k.
Question Is there a subset $C' \subseteq C$ of size at most k, such that c is the unique $\mathcal{E}$ winner of the election $(C - C', V)$?

We can also define the destructive case $\mathcal{E}$-Destructive Control by Deleting Candidates ($\mathcal{E}$-DC-DC) analogously. Here we ask whether it is possible to keep candidate $c \notin C'$ from being the unique $\mathcal{E}$ winner of the election $(C - C', V)$.

The following notions are due to Bartholdi, Tovey, and Trick [4]. Let $\mathfrak{CT}$ be a control type. If the chair can never change a non winner to a unique winner in election $\mathcal{E}$ by exerting control of type $\mathfrak{CT}$, we say that $\mathcal{E}$ is *immune to* $\mathfrak{CT}$. If a voting system $\mathcal{E}$ is not immune to $\mathfrak{CT}$, then it is said to be *susceptible to* $\mathfrak{CT}$. If a voting system $\mathcal{E}$ is susceptible to $\mathfrak{CT}$, and the chair's task of controlling the election is NP-hard, $\mathcal{E}$ is said to be *resistant to* $\mathfrak{CT}$. If a voting system $\mathcal{E}$ is susceptible to $\mathfrak{CT}$, and the corresponding decision problem can be solved in polynomial time, the voting system is said to be *vulnerable to* $\mathfrak{CT}$. Table 1 shows the previous results on control complexity of Copeland and Llull, see [13].

In this paper we consider optimization versions of these decision problems, which look for a subset of candidates of minimum size which have to be removed or added in order to control the election. For the voting systems $\mathcal{E} \in$ {Copeland, Llull} we introduce the following four control optimization problems.