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Constructing independent spanning trees with height n on the n-dimensional crossed cube

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HIGHLIGHTS

- Give the parallel construction of $n-2$ ISTs rooted at 0 with height $\leq n$ on CQ_n for $n \geq 4$.
- Present a binary XOR operation to obtain the ISTs rooted at any node similar to 0.
- Design recursive construction of $n-2$ ISTs rooted at any node with height $\leq n$.
- Discuss similar parallel construction of ISTs on locally twisted cube and hypercube.

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ABSTRACT

Independent spanning trees (ISTs) on networks have applications in reliable broadcasting, reliable communication protocols, secure message distribution. Towards the ISTs on crossed cubes, although several results have been obtained, the maximum height of the ISTs on the n-dimensional crossed cube CQ_n is no less than $n+1$ for $n \geq 3$. So we have the question whether there exist multiple ISTs with height lower than $n+1$ on CQ_n. In this paper, firstly, we present the construction of $n-2$ ISTs rooted at node 0 on CQ_n, the maximum height of which is no more than n for $n \geq 4$. A parallel algorithm with the time complexity $O(N)$ is also presented, where $N = 2^n$. Secondly, we present a binary XOR operation to transform the above $n-2$ ISTs into the $n-2$ ISTs rooted at any node that is similar to 0 on CQ_n. Thirdly, we revise the recursive algorithm CIST from Cheng et al. (2013) to construct $n-2$ ISTs rooted at any node on CQ_n with the maximal height n, where the time complexity is $O(N \log^2 N)$. We have also extended the efficient parallel method to locally twisted cubes.

1. Introduction

Interconnection networks have applications in many aspects, such as parallel machines [1], data center networks [2], network-on-chips [3,4], etc. Hypercube is a well-known interconnection network that possesses many attractive properties such as lower diameter, high connectivity, and symmetry. As an important variant of the hypercube, the crossed cube has the good properties such as the diameter, wide diameter, and fault-diameter are all about half to those of the hypercube with the same dimension, respectively [5,6]. As a consequence, many results have been obtained by researchers [5–12].

An interconnection network can be abstracted as a graph G, where $V(G)$ denotes the node set and $E(G)$ denotes the edge set. Given a graph G, a set of spanning trees rooted at a node u on G are independent spanning trees (ISTs for short) if for any node $v \in V(G) \setminus \{u\}$, the paths in these ISTs joining u and v have neither node nor edge in common, except u and v. ISTs have various applications in reliable communication protocols, reliable broadcasting, and secure message distribution [13]. As to the existence of ISTs on graphs, there exists a conjecture that an n-connected graph $G(n \geq 1)$ has n ISTs rooted at an arbitrary node on G. However, the conjecture has only been proved to be true for graphs with $n \leq 4$ [14–16]. So far, the construction of ISTs on special graphs for $n \geq 5$ is still the focus of attention of researchers. The above conjecture has been proved true for many special graphs, such as planar graphs [17,18], product graphs [19], multidimensional torus networks [20], hypercubes [21], crossed cubes [22,23], Möbius cubes [24,25], parity cubes (or twisted-cubes) [26,27], even graphs [28], odd networks [29], enhanced hypercubes [30], RTCC-Pyramids [31], and etc. Towards the ISTs on crossed cubes, [22] and [23] gave a recursive algorithm and a parallel algorithm to

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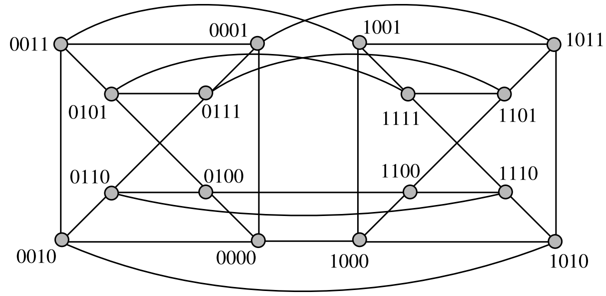

Fig. 1. 4-dimensional crossed cube CQ_4.

construct n ISTs rooted at any node on n-dimensional crossed cube CQ_n, respectively.

For the ISTs, the height is an important performance measure, that is, reducing the height of a spanning tree rooted at the source node is useful to design an efficient broadcasting scheme [32]. For crossed cubes, the maximum height of the trees in the set of ISTs on CQ_n obtained in the literature is no less than $n+1$. Noticing that the maximum height of the ISTs constructed on hypercubes also has the height $n+1$ [21] and the crossed cube has only about half of the diameter to that of the hypercube with the same dimension, do there exist multiple ISTs with the maximum height lower than $n+1$? We will try to answer this question in this paper.

2. Preliminaries

The n-dimensional crossed cube CQ_n has 2^n nodes. Each node of CQ_n is represented by a unique binary string with length n, called the address of the node. For example, node u is represented by $u_{n-1}u_{n-2}\dots u_0$. The binary bit u_{n-1} is called the most significant bit of the address of node u. Here, we can also use decimal numbers to denote the addresses of the nodes in CQ_n. Suppose that V' is a nonempty subset of $V(G)$. We use $G[V']$ to denote the subgraph of G induced by V'. The union of graphs G_1 and G_2, written as $G_1 \cup G_2$, has node set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$. We adopt the definition of CQ_n as follows.

Definition 2.1 (*[1,6]*)**.** Two binary strings $x = x_1x_0$ and $y = y_1y_0$ of length two are said to be *pair-related* (denoted by $x \sim y$) if and only if $(x, y) \in \{(00, 00), (10, 10), (01, 11), (11, 01)\}$.

Definition 2.2 (*[1]*)**.** The n-dimensional crossed cube CQ_n is recursively defined as follows. CQ_1 is the complete (undirected) graph on two nodes whose addresses are 0 and 1. CQ_n consists of two subcubes CQ^0_{n-1} and CQ^1_{n-1}. The most significant bit of the addresses of the nodes in CQ^0_{n-1} and CQ^1_{n-1} are 0 and 1, respectively. The nodes $u = u_{n-1}u_{n-2}\dots u_0 \in CQ^0_{n-1}$ and $v = v_{n-1}v_{n-2}\dots v_0 \in CQ^1_{n-1}$, where $u_{n-1} = 0$ and $v_{n-1} = 1$, are joined by an edge in CQ_n if and only if

(1) $u_{n-2} = v_{n-2}$ if n is even, and
(2) $u_{2i+1}u_{2i} \sim v_{2i+1}v_{2i}$, for $0 \le i < \lfloor \frac{n-1}{2} \rfloor$.

Fig. 1 shows the 4-dimensional crossed cube CQ_4. For a node $v \neq u$ in T rooted at node u on CQ_n, we define parent(v, T) as the parent of node v and ancestor(v, T) as the set of nodes in the path from parent(v, T) to u in T. We follow the definitions of d-neighbor from [22]. $N(u, d)$ is used to denote d-neighbor of u, $N(W, d) = \{N(w, d) | w \in W\}$ for $W \subset V(CQ_n)$ denotes the set of the d-neighbors of all nodes in W.

3. Constructive algorithm of ISTs on CQ_n

In what follows, firstly, we will present an efficient algorithm to construct $n-2$ ISTs rooted at node 0 with the height no more than n on CQ_n. As to the nodes in CQ_n, Kulasinghe et al. proved that CQ_n is not node-transitive in [7] and showed that the nodes in CQ_n can be divided into $2^{\lceil (n-4)/2 \rceil}$ equivalence classes in [8]. Then, we further show that the ISTs rooted at any node similar to 0 can be obtained by transforming the ISTs rooted at node 0. At last, we revise the recursive algorithms from [22] to construct ISTs rooted at any node on CQ_n with the height no more than n and we also show that the time complexity is $O(N \log^2 N)$, where $N = 2^n$.

3.1. Constructive algorithm of ISTs rooted at node 0 on CQ_n

By the result in [22], we can obtain n ISTs rooted at any node on CQ_n with the height $n+1$ for $n \ge 2$. For example, the four ISTs rooted at node 0 on CQ_4 are shown in Fig. 2. The height of any of the four ISTs is 5. However, we can construct two ISTs rooted at node 0 with the maximum height 3 on CQ_4, which are shown in Fig. 3.

Based on Definition 2.2, we present an algorithm, called HT, to construct a tree on CQ^1_{n-1} isomorphic to the tree rooted at node 0 on CQ^0_{n-1}.

Notation 1. *We use η to denote the isomorphic mapping from T_1 to T_2 defined by Algorithm HT.*

Algorithm 1 HT(T_1, n)

Input: An integer n and a tree T_1 rooted at node 0 on CQ^0_{n-1} for $n \ge 1$.
Output: A graph T_2 on CQ^1_{n-1}.
Begin
1: Construct tree T_2 by adding 2^{n-1} to each node in T_1;
End

Lemma 3.1. *Given a tree T_1 rooted at node 0 on CQ^0_{n-1} for $n \ge 1$, the graph T_2 obtained by Algorithm HT is a tree rooted at node 2^{n-1} on CQ^1_{n-1}.*

Proof. By Definition 2.2, it is obvious that T_2 obtained by Algorithm HT is a tree rooted at node 2^{n-1} on CQ^1_{n-1}. □

Based on Lemma 3.1, we have the following corollary.

Corollary 3.1. *Suppose that T_1 is a spanning tree rooted at node 0 on CQ^0_{n-1} for $n \ge 1$. The graph T_2 obtained by Algorithm HT is a spanning tree rooted at node 2^{n-1} on CQ^1_{n-1}.*

Based on Lemma 3.1 and Algorithm HT, we have the following lemma.

Lemma 3.2. *Suppose that the root node 0 in T_1 on CQ^0_{n-1} has two or more children, denoted as $v_1, v_2, \dots, v_m$, where $2 \le m \le n-1$. Then, the root node 2^{n-1} in T_2 on CQ^1_{n-1} has children $v_1 + 2^{n-1}$, $v_2 + 2^{n-1}$, …, $v_m + 2^{n-1}$.*

Based on Algorithm HT, T_2 constructed by Algorithm HT is isomorphic to T_1. Recalling that η is the isomorphic mapping from T_1 to T_2 provided by Algorithm HT, we further give the definitions of IENS with respect to the tree rooted at node 0 and IENSP as follows.

Definition 3.1. For any integer n with $n \ge 5$, suppose that T_1 is a spanning tree rooted at node 0 on CQ^0_{n-1}. If there exists a nonempty set $W \subseteq V(T_1) \setminus \{0\}$ such that