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Model matching of input/output asynchronous sequential machines based on the semi-tensor product of matrices

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HIGHLIGHTS

- The model matching process of the input/output asynchronous sequential machines is considered with the minimal state set of the controller machine, which reduces the computational complexity.
- In the framework of the semi-tensor product of matrices, three polynomial algorithms are proposed for the design of the controller of the model matching problem and these algorithms only involve matrix operation.
- The dynamics of the composed system are converted to a discrete event bilinear system equation and can be used to verify the results of the model matching problem easily.

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ABSTRACT

This paper investigates the model matching problem of input/output asynchronous sequential machines (ASMs) based on the semi-tensor product (STP) of matrices. The controller of the model matching problem is separated to an observer B and a control unit F. When the controller $C = (F, B)$ of the model matching problem exists, the control process is considered with the minimal state set of the controller. Three algorithms which only involve matrix operations are proposed to assign values to the structure matrices of the observer B and the control unit F. In the framework of the STP of matrices, a new algebraic expression for the dynamics of the separated closed-loop system is established and we can easily verify the results of the model matching problem. In this paper, the model matching process is considered with matrix multiplication which is easy to operate. An illustrative example is presented to show the theoretical results and the effectiveness of the proposed algorithms.

1. Introduction

Asynchronous sequential machines (ASMs) are digital logic circuits in which states change without clocking [1]. With the character of clockless operation, the states of ASMs can change very fast [2]. ASMs have played important role in high-speed computing systems [3], parallel computation systems [4,5], processes of molecular biology [6], and numerous other fields.

There are many literatures that have investigated ASMs, such as input/output control of asynchronous machines with races [7], state feedback control of asynchronous machines with nondeterministic models [8], corrective control of input/output asynchronous sequential machines with adversarial inputs [9], and many others. From these literatures, we can find that many control problems of ASMs can be converted to model matching problem. So it has great significance to investigate the model matching problem of ASMs.

Model matching problem is to design a feedback controller for a given machine so that the response of the closed-loop system matches that of a reference model. Model matching problem has been widely studied [10–13]. For the model matching problem of ASMs, the main theoretical results were proposed in [14,15]. The literature [15] has given the necessary and sufficient conditions for the existence of the controller of input/output ASMs, when the controllers exist, it also proposed algorithms to design them. However, there are some deficiencies about those results. On the one hand, the state set of the controller is too large and needs

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additional steps to reduce the state set. On the other hand, the literature expressed the matching results by a series of state transition diagrams and the operation of the matching process is tedious.

Matrix technology provides an elegant tool for discrete event system. By resorting to the STP of matrices, [16] proposed a matrix-based approach for the model matching problem of input/state ASMs. The STP of matrices was proposed by Cheng in [17] which is a generalization of the conventional matrix product. The literature [18] have given a brief introduction about the definitions and the basic properties of the STP of matrices. The STP of matrices has been successfully applied to many fields, such as Boolean networks [19], Petri Nets [20] and finite state machines [21]. Moreover, the STP of matrices can be extended to the complex systems and networks, which have been widely used in public hygienics, mathematics and computer science [22,23].

Literature [16] only considered the input/state ASMs, and the approach proposed in it cannot be used to solve the model matching problem of input/output ASMs. In this paper, we will consider the model matching problem of input/output ASMs based on the theoretical results proposed in [15], which is more complicated than input/state ASMs. Moreover, the results proposed in this paper can be also applied to the model matching problem of input/state ASMs. Using the STP of matrices, we derive the algebraic expression for the dynamics of the separated closed-loop system. The states set of the control unit F in [15] is redundancy and needs to be reduced. In this paper, we will directly use the minimal state set which is equal to the reduced state set in [15] to design the control unit F.

The main contributions of this paper are as follows.

(1) Design the model matching process with the minimal state set of the control unit F, which reduces the computational complexity.
(2) Three algorithms are proposed to assign values to the structure matrices of the observer B and the control unit F. Moreover, these algorithms only involve the operations of matrices.
(3) A new algebraic expression for the dynamics of the separated closed-loop system are established. By calculating the algebraic equations, we can get the state transitions and outputs of the system and then we can easily verify the results of the model matching problem.

This paper is organized as follows. Section 2 introduces preliminaries of the STP of matrices and the input/output ASMs. Section 3 is divided into four parts: First, we show the matrix representation of input/output ASMs; second, we consider the model matching process with the minimal states of the observer B and the control unit F; third, three algorithms are proposed to assign values to the structure matrices of the observer B and the control unit F; finally, we give the algebraic expression for the dynamics of the separated closed-loop system. In Section 4, an illustrative example is presented to show the effectiveness and the application of the theoretical results proposed in Section 3.

2. Preliminaries

2.1. Notations

In this subsection, we introduce some notations, which will be used in the sequel.

- $\mathcal{M}_{m\times n}$ is the set of $m \times n$ real matrices.
- $\mathbb{R}^n$ is the set of n dimensional column vectors.
- $Col_j(M)$ is the jth column of the matrix M and we denote $Col(M)$ as the set of columns of matrix M.
- $\delta_n^j := Col_j(I_n)$, where I_n is the identity matrix. Let $\delta_n^0 := [\underbrace{0, 0, \dots, 0}_{n}]^T$.
- $\Delta_n := \{\delta_n^1, \dots, \delta_n^n\}$; $\tilde{\Delta}_n := \{\delta_n^0, \delta_n^1, \dots, \delta_n^n\}$.
- $1_n \in \mathbb{R}^n$ is the vector whose elements all equal to 1.
- $M \in \mathcal{M}_{m\times n}$ is called a logical matrix (resp., generalized logical matrix) if $Col(M) \subseteq \Delta_m$ (resp., $Col(M) \subseteq \tilde{\Delta}_m$). We denote the set of $m \times n$ logical matrices (resp., generalized logical matrices) by $\mathcal{L}_{m\times n}$(resp., $\tilde{\mathcal{L}}_{m\times n}$).
- If $M \in \mathcal{L}_{m\times n}$(resp., $\tilde{\mathcal{L}}_{m\times n}$), then it can be expressed as $M = [\delta_m^{i_1}, \dots, \delta_m^{i_n}]$ and its shorthand form, for simplicity, is $M = \delta_m[i_1, i_2, \dots, i_n]$, where $i_k \in \{0, 1, \dots, m\}$, $k = 1, 2, \dots, n$.

2.2. Semi-tensor Product (STP) of Matrices

In this subsection, we introduce some preliminaries about the STP of matrices, which will be used in this paper. For more details on STP, we can refer to [17].

Definition 1 (*[17]*). Let $A \in \mathcal{M}_{m\times n}$, $B \in \mathcal{M}_{p\times q}$. Then the semi-tensor product (STP) of A and B is defined as

$$A \ltimes B = (A \otimes I_{t/n})(B \otimes I_{t/p}) \tag{1}$$

where $t = lcm(n, p)$ is the least common multiple of n and p. $\otimes$ is the Kronecker product.

Remark 1. The STP is a generalization of the conventional matrix product, and when $n = p$, we have $A \ltimes B = AB$. Throughout this paper the matrix product is assumed to be the STP. We usually omit the symbol "$\ltimes$" hereinafter.

Definition 2 (*[17]*). A swap matrix $W_{[m,n]}$ is a $mn \times mn$ matrix, which is defined as

$$\begin{aligned} W_{[m,n]} = \delta_{mn}\,[&1, m+1, \dots, (n-1)m+1, \\ &2, m+2, \dots (n-1)m+2, \dots \\ &m, 2m, \dots, nm]\,. \end{aligned} \tag{2}$$

Remark 2. When $m = n$, we denote $W_{[m,n]} = W_{[n]}$.

Lemma 1 (*[17]*). *Let $X \in \mathbb{R}^m$ and $Y \in \mathbb{R}^n$ be two column vectors. Then $W_{[m,n]}XY = YX$, $W_{[n,m]}YX = XY$.*

Lemma 2 (*[17]*). *Assume $A \in \mathcal{M}_{m\times n}$ is given.*
(1) Let $X \in \mathbb{R}^t$ is a row vector. Then $AX = X(I_t \otimes A)$.
(2) Let $X \in \mathbb{R}^t$ is a column vector. Then $XA = (I_t \otimes A)X$.

Lemma 3. *Let $\Phi_n = diag[\delta_n^1, \delta_n^2, \dots, \delta_n^n]$ be a power-reducing matrix. Then*

$$X^2 = \Phi_n X \tag{3}$$

where $X \in \Delta_n$.

Lemma 4. *Let $\delta_{m_1}^{j_1} \ltimes \delta_{m_2}^{j_2} \ltimes \cdots \ltimes \delta_{m_t}^{j_t} = \delta_{m_1\times m_2\times\cdots\times m_t}^{l}$ be the STP of matrices of t logical vectors. Then we have $\delta_{m_i}^{j_i} = S_i \ltimes \delta_{m_1\times m_2\times\cdots\times m_t}^{l}$, $(i = 1, 2, \dots, t)$, where*

$$\begin{cases} S_1 = I_{m_1} \otimes 1_{m_2\times\cdots\times m_t}, \\ S_2 = \underbrace{\left[I_{m_2} \otimes 1_{m_3\times\cdots\times m_t}, \dots, I_{m_2} \otimes 1_{m_3\times\cdots\times m_t}\right]}_{m_2}, \\ \vdots \\ S_{t-1} = \underbrace{\left[I_{m_{t-1}} \otimes 1_{m_t}, \dots, I_{m_{t-1}} \otimes 1_{m_t}\right]}_{m_2\times\cdots\times m_{t-1}}, \\ S_t = \underbrace{\left[I_{m_t}, \dots, I_{m_t}\right]}_{m_2\times\cdots\times m_t}. \end{cases} \tag{4}$$