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ORIGINAL ARTICLE

A New Characterisation of the Minimal Solution Set to Max-min Fuzzy Relation Inequalities

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Abstract In this paper, max-min fuzzy relation inequalities are applied to describe a BitTorrent-like Peer-to-Peer file sharing system. In order to decrease the network congestion under some fixed priority grade of the terminals, we define concept of lexicographic solution in such system. It is found that each minimal solution of max-min fuzzy relation inequalities happen to be a lexicographic solution. Moreover, the lexicographic solution set is exactly the minimal solution set. Numerical example is given to illustrate our results. At last, for the addition-min fuzzy relation inequalities, we provide a guess, trying to describe the relation between its lexicographic solution set and minimal solution set.

Keywords Fuzzy relation inequality · Fuzzy relation equation · Lexicographic solution · Minimal solution · Max-min composition · Peer-to-Peer network system

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1. Introduction

It is well known that the fuzzy relation equation was proposed by Sanchez [1] for the first time. Since then, many scholars shown their interests in the investigation of fuzzy relation equations or inequalities as well as its corresponding optimization

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problems. Both theoretical results [2-7] and practical applications [8-10] of the fuzzy relation system could be found in the existing works. Accompanied with research on fuzzy relation equations or inequalities, the corresponding optimization problems [11-17] were also investigated in recent years.

The solution set of a fuzzy relation system is fully determined by the unique maximum solution and a number of minimal solutions. Because of the special structure of the solution set, solving a fuzzy relation system is usually equivalent to searching all its minimal solutions. In the past period, various methods to the minimal solution set appeared. It has become an important issue in this research field. However, the existing methods seem to be inefficient [18]. Solving all the minimal solutions of a system of fuzzy relation equations (or inequalities) with max-min [19-20] or max-product [21-22] composition is an NP-hard problem, equivalent to a set covering problem. It remains a challenging problem until now.

Recently, a new type of fuzzy relation inequality (FRI), composed by the so-called addition-min operator, was proposed by Li et al. [23] to describe the quantitative relation of a Peer-to-Peer (P2P) file sharing system under BitTorrent-like transmission mechanism. In Ref. [23-25], the authors considered the total quality requirement of download traffic of the terminals. In this situation, the P2P file sharing system was reduced into the addition-min FRIs. However, as shown in [26], when considering the highest quality requirement of download traffic of the terminals, the P2P file sharing system could be characterized by max-min fuzzy relation equations or inequalities. Formulae of max-min FRIs are as follows:

$$\vec{a}_i \circ \vec{x} \geq b_i, \quad i = 1, 2, \dots, m, \tag{1}$$

where $\vec{a}_i = (a_{i1}, a_{i2}, \dots, a_{in}) \in [0, 1]^n$, $\vec{x} = (x_1, x_2, \dots, x_n) \in [0, 1]^n$, $b_i \in [0, 1]$, and

$$\vec{a}_i \circ \vec{x} = (a_{i1} \wedge x_1) \vee (a_{i2} \wedge x_2) \vee \dots \vee (a_{in} \wedge x_n),$$

$i = 1, 2, \dots, m$. Besides, if we denote

$$\vec{a}_i \oplus \vec{x} = a_{i1} \wedge x_1 + a_{i2} \wedge x_2 + \dots + a_{in} \wedge x_n,$$

then the addition-min FRIs may be written as

$$\vec{a}_i \oplus \vec{x} \geq b_i, \quad i = 1, 2, \dots, m, \tag{2}$$

where all the parameters means the same as those in system (1). The matrix form of (1) is $A \circ x \geq b$, while that of (2) is $A \oplus x \geq b$.

In order to decrease the network congestion in the P2P file sharing system under different priority grade (degree of importance) of the terminals, Yang et al. [27] proposed the lexicographic minimum solution of system of addition-min FRIs with effective resolution algorithm and illustrative numerical example. Considering the similar application background, we define the lexicographic (minimum) solution of system of max-min FRIs in this paper. The main contribution of this study is to

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