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ORIGINAL ARTICLE

On Strong Intervals in Fuzzy Graphs

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Abstract Intervals and convexity play crucial roles in the applications of graph theory such as town planning and design of graphics. In this article, the concept of geodetic interval in graphs is extended to fuzzy graphs. Intervals are useful in the study of properties of fuzzy graphs which depend on the geodetic distance between vertices. The axiomatic definition of intervals in fuzzy graphs are used to define intervals in different fuzzy graph structures like fuzzy trees and complete fuzzy graphs. Finally a set theoretic operations of intervals like union, intersection are also discussed and some results are obtained.

Keywords Geodesic distance · Fuzzy tree · Interval function · Strong path

1. Introduction

In 1980 Mulder [7] introduced interval function of a graph and showed that interval function is a helpful tool to study different metric properties of connected graphs. Later, along with Nebesky, he gave the axiomatic characterization for interval functions in [8].

In [3], it was shown that if a fuzzy graph H is connected, then there is a strong path between any two vertices of H. A strong path ρ from x to y is called a geodesic if there is no shorter strong path from x to y [2]. For any vertex x, the singleton $\{x\}$

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is called a geodesic from x to x. The length of a geodesic from x to y is called the geodetic distance from x to y, denoted by $d_g(x, y)$ [2].

Fuzzy graph theory is a subject of interest today, having a large number of researchers around the globe. Several authors including Bhutani and Rosenfeld [2, 3], Mathew and Sunitha [4, 5], Morderson [6], Rosenfeld [9], Sunitha and Vijayakumar [13], Sameena and Sunitha [10, 11, 12], Akram and Dudek [1] have introduced several concepts in fuzzy graphs.

In this article, we introduce strong intervals in fuzzy graphs. We use axiomatic definition of intervals in fuzzy graphs to establish different properties of such intervals. Also set-theoretic operations of intervals in fuzzy graphs are studied towards the end.

2. Preliminaries

A fuzzy graph (f-graph) [9] is a pair $G = (\sigma, \mu)$ where σ is a fuzzy subset of a set S and μ is a fuzzy relation on σ. We assume that S is finite and nonempty, μ is reflexive and symmetric [9]. In all the examples σ is chosen suitably. Also, we denote the underlying crisp graph by $G^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in S/ \ \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in S \times S/ \ \mu(u, v) > 0\}$. A fuzzy graph $H = (\tau, \nu)$ is called a partial fuzzy subgraph of $G = (\sigma, \mu)$ if $\tau(u) \leq \sigma(u)$ for every $u \in \tau^*$ and $\nu(u, v) \leq \mu(u, v)$ for every $(u, v) \in \nu^*$. A path P of length n is a sequence of distinct vertices $u_0, u_1, \cdots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, \ \ i = 1, 2, \cdots, n$ and the degree of membership of a weakest edge is defined as its strength [5]. If $u_0 = u_n$ and $n \geq 3$, then P is called a cycle and a cycle P is called a fuzzy cycle (f-cycle) if it contains more than one weakest edge [6]. The strength of connectedness between two vertices x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$ [5]. An $x - y$ path P is called a strongest $x - y$ path if its strength equals $CONN_G(x, y)$ [9]. An f-graph $G = (\sigma, \mu)$ is connected if for every $x, y \in \sigma^*$, $CONN_G(x, y) > 0$. Throughout, we assume that G is connected. An edge of a fuzzy graph is called strong if its weight is at least as great as the strength of connectedness of its end vertices when it is deleted and an $x - y$ path P is called a strong path if P contains only strong edges [2, 3].

An edge is called a fuzzy bridge (f-bridge) of G if its removal reduces the strength of connectedness between some pair of vertices in G. Similarly a fuzzy cutvertex (f-cutvertex) w is a vertex in G whose removal from G reduces the strength of connectedness between some pair of vertices other than w. A connected f-graph $G = (\sigma, \mu)$ is a fuzzy tree (f-tree) if it has a partial fuzzy spanning subgraph $F = (\sigma, \nu)$, which is a tree, where for all edges (x, y) not in F there exists a path from x to y in F whose strength is more than $\mu(x, y)$. Note that F is the unique maximum spanning tree (MST) of G [13]. A complete fuzzy graph (CFG) is an f-graph $G = (\sigma, \mu)$ such that $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ for all x and y.

3. Intervals in Fuzzy Graphs

In this section, we introduce the concept of intervals in fuzzy graphs. Henry Martin Mulder has introduced the concept of interval function in a graph as follows.