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ORIGINAL ARTICLE

The Fixed Point Property for Intuitionistic Fuzzy Lattices

Lemnaouar Zedam · Soheyb Milles · Ewa Rak

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Abstract In this paper, based on the concept of intuitionistic fuzzy lattice previously introduced by Tripathy and his colleagues, a class of intuitionistic fuzzy complete lattices is proposed with some interesting characterizations given. In particular, we show the fixed point property for this proposed class. Conversely, we show that any intuitionistic fuzzy lattice is complete having its fixed point property. These results establish a criterion for completeness of intuitionistic fuzzy lattices in terms of the fixed points of their intuitionistic fuzzy monotone mappings.

Keywords Atanassov's intuitionistic fuzzy set · Intuitionistic fuzzy relation · Intuitionistic fuzzy lattice · Fixed point property

1. Introduction

The fixed point property is one of the most famous topics in order theory. Let P be an ordered set, P is said to have the fixed point property if every order-preserving

Corresponding Author: Lemnaouar Zedam (✉)
Laboratory of Pure and Applied Mathematics, Department of Mathematics, Med Boudiaf University of Msila, P.O. Box 166 Ichbilia, M'sila 28105, Algeria
email:l.zedam@gmail.com
Soheyb Milles (✉)
Laboratory of Pure and Applied Mathematics, Department of Mathematics, Med Boudiaf University of Msila, P.O. Box 166 Ichbilia, M'sila 28105, Algeria
Ewa Rak (✉)
Faculty of Mathematics and Natural Sciences, University of Rzeszów, Poland
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map f of P into itself has a fixed point. One of the important roles of the fixed point property is to combine properties of a set with the properties of maps in that set. For the new structures the fixed point property can provide a familiar setting, helping us to investigate these structures.

Atanassov [1] introduced a new structure called intuitionistic fuzzy set or Atanassov's intuitionistic fuzzy set as it is also called by several authors. Atanassov's intuitionistic fuzzy set is a generalization of Zadeh's fuzzy set previously introduced in [27]. In fuzzy set theory, the non-membership degree of an element x can be viewed as $\nu_A(x) = 1 - \mu_A(x)$ (using the standard strong negation) and this is fixed. While in the intuitionistic fuzzy setting, the non-membership degree is a more-or-less independent degree: the only condition is that $\nu_A(x) \leq 1 - \mu_A(x)$. Certainly, fuzzy sets are Atanassov's intuitionistic fuzzy sets by setting $\nu_A(x) = 1 - \mu_A(x)$.

Based on Atanassov's intuitionistic fuzzy set, Burillo and Bustince [9, 10] introduced the concept of intuitionistic fuzzy relation, particulary, they introduced the intuitionistic fuzzy order or (intuitionistic fuzzy ordered set) as a natural generalization of fuzzy order relation previously introduced by Zadeh [28]. Intuitionistic fuzzy relations theory has been applied to many different fields, such as decision making, mathematical modeling, medical diagnosis, control systems, machine learning, market prediction, and so on.

One of the important problems of fuzzy and intuitionistic fuzzy ordered set is to obtain an appropriate concept of the particular elements on such structure, like upper bound, maximum, supremum, maximal elements and their duals, in order to obtain new structures and particular classes of fuzzy and intuitionistic fuzzy ordered sets. Several theoretical and applicational results connected with this problem can be found, e.g. in Bělohlávek [6], Bodenhofer and Klawonn [7], Bustince and Burillo [12, 13], Coppola et al. [14], Tripathy et al. [24] and Zadeh [28]. In particular, Tripathy and his colleagues [24] introduced the concepts of the upper bound, the supremum and their duals of subsets on a universe X with respect to an intuitionistic fuzzy order defined on it. Also, they introduced and studied a concept of lattice with respect to an intuitionistic fuzzy order defined on it. This concept is extensively used and discussed in the fuzzy and intuitionistic fuzzy settings by several authors [6, 19, 20, 23, 25, 31].

In a recent conference paper [21], based on the concept of intuitionistic fuzzy lattice previously proposed by Tripathy et al. [24], we introduced the notion of intuitionistic fuzzy complete lattice and investigated its basic characterizations. The introduced notion of intuitionistic fuzzy complete lattice is a generalization of the notion of crisp complete lattice. In this paper, we extend these characterizations by considering others completeness criterions. The characterizations of intuitionistic fuzzy complete lattices expressed in terms of the existence of the supremum or the infimum of their subsets, in terms of intuitionistic fuzzy chains and maximal chains and in terms of intuitionistic fuzzy ascending (resp. descending) chains are given. Furthermore, we will show that an intuitionistic fuzzy lattice X is complete if and only if any intuitionistic fuzzy monotone mapping $f : X \to X$ has a fixed point, i.e., an intuitionistic fuzzification of Tarski-Davis's fixed point theorem [15, 22]. This leads to see clearly that any intuitionistic fuzzy complete lattice has the fixed point property and vice versa. These results show the key role of the fixed point property