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ORIGINAL ARTICLE

Dynamical Study in Fuzzy Threshold Dynamics of a Cholera Epidemic Model

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Abstract In this paper, a fuzzy mathematical model on cholera disease has been developed in which all parameters related to the disease have been considered as fuzzy numbers. Here, total human population is divided into three subpopulations such as susceptible persons, infected ones and recovered ones. Also, the bacterial population is the Vibrio Cholerae in the environment. Then the existence condition and boundedness of solution to our proposed mathematical model have been discussed. Also, the different equilibrium points and the stability condition of the system around these equilibrium points have been analyzed. The global stability condition of the proposed system around the endemic equilibrium point has been also discussed. Finally, some numerical simulations have been shown to test the theoretical results of the system.

Keywords Cholera disease · Fuzzy number · Triangular Fuzzy number · UFM method · Local stability · Global stability

1. Introduction

Cholera is an acute diarrhoeal infection caused by ingestion of food or water contaminated with the bacterium Vibrio Cholerae. It causes mortality, disability, social and economic damage for millions of people in the whole world specially in developing

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countries. So, it is a major threat to human beings. Two of the toxigenic V. Cholerae $O1$ and $O139$ are free-living bacterial organisms found in fresh and briny water. Frequently, they are found in association with zooplankton, shellfish and aquatic plants. It spreads through the contaminated food, drinking water and also from the feces of infected human. The main symptoms of cholera are watery diarrhea, vomiting, rapid dehydration, rapid heart rate, loss of skin elasticity, dry mucous membranes etc. Severe outbreaks usually occur in underdeveloped areas with inadequate sanitation, poor hygiene and limited access to safe water supplies. Although there are many recent progresses in medical sciences, cholera remains now as a global threat in some parts of the World.

Mathematical models have become more important tools for analyzing the spread of cholera disease. Basically, ordinary differential equation is used for formulation of this type of problem and provides some mathematical answer and explanation. A crisp mathematical model on cholera disease was described by Capasso [9]. It was consisted with two equations to follow the dynamics of infected individuals and the number of free-living infective stages. More recently Codeco [10] developed a more general model of cholera with an additional equation in the population. Modeling and analysis of the spread of carrier dependent infectious diseases with environmental effects was explored by Singh et al. [7]. There exist many mathematical models in crisp environment on cholera disease which explore the spread and control strategies of the disease such as [8, 11, 12, 14 – 20]. In 2016, Misra et al. [22] studied the effects of bacteriophage infection on the cholera disease dynamics. After that, Soufiane and Touaoula [26] investigated an epidemic model with infection age with the help of a set of nonlinear differential equation. Also, Nasr-Azadani et al. [23] explored the impact of climate change on cholera disease dynamics in the Ganges-Brahmaputra basin. Again, in 2017, Sun et al. [21] studied a mathematical model on cholera with different types of control strategies. After that, Cai et al. [24] and Lemos-Paio et al. [25] have investigated the cholera epidemic model in the presence of vaccination and treatment as control parameter.

The parameters involved in the mathematical models on cholera disease discussed above are crisp in nature. But it is found that the biological parameters involve in the differential equations are not always fixed. In the real world every community is changing with the varying environments. In the present time, the global warming is the main problem in the whole globe. It is the increase of earth's average surface temperature due to effect of greenhouse gases such as carbon dioxide emissions from burning fossil fuels or from deforestation which trap heat that would otherwise escape from earth. The change of temperature strongly effects on the reproduction rate of the bacterial population. Many parameters may oscillate with the change of environments in real world ecosystem. These parameters are also varying due to both natural and human activities such as earthquake, climate warming, financial crisis etc. Therefore, the interactions between the human and bacteria and the dynamics of c-holera disease may be influenced by the environmental variations. In this respect, the fuzzy mathematical model is more meaningful than the crisp model. So, in this paper, we have used fuzzy set theory [5] to formulate this cholera model. There exist very few number of papers in infectious disease model in fuzzy environment [13]. The