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ORIGINAL ARTICLE

Multi Fuzzy Fractal Theorems in Fuzzy Metric Spaces

B. Prasad · K. Katiyar

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Abstract The Hutchinson-Barnsley (HB) operators are studied for multi iterated function system in a general setting. Some existence and uniqueness results are obtained in fuzzy fractal and multi fuzzy fractal spaces. From them, we obtain our results, which generalize some recent results in the literature.

Keywords Fractal space · Fuzzy metric space · Multi fuzzy metric space · Multi fuzzy fractal space

1. Introduction

Zadeh [28] introduced fuzzy sets in order to study and represent the degree of uncertainty in a purely mathematical and formal way. Since then, researchers have been exploring different domains of knowledge in the framework of this new notion. It is considered to be of prime importance in various scientific and engineering applications involving imprecision variables. The concept of fuzzy metric space was introduced by a number of authors in different manners. Kramosil and Michalek [18] were the first to advance fuzzy metric space to measure uncertainty in measuring distance between two objects or sets. George and Veeramani [12-13] modified their concept by defining a Hausdorff and countable topology on the space. Thereafter, a lot of

B. Prasad (✉)
Department of Mathematics, Jaypee Institute of Information Technology, Noida 201307, India
email: b_prasad10@yahoo.com
K. Katiyar
Department of Mathematics, Jaypee Institute of Information Technology, Noida 201307, India
email: kuldipkatiyar.jiitn@gmail.com

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work is reported in this direction under different settings (see [11, 14-16]). Mihet [20] obtained fuzzy version of the celebrated Banach contraction principle. Lopez and Romaguera [23] studied several important properties of Hausdorff fuzzy metric on compact sets. Qui and Zhang [4] established that for a given positive continuous t-norm there exists fuzzy metric space for which the given t-norm is the strongest one. In [5, 7], Qui et al. obtained interesting fixed points theorems in fuzzy metric spaces and Qui et al [6] generalized the classical Hausdorff metric with t-norm and studied its basic properties. The concept of fuzziness is also extended to fractals, a new frontier of science, initiated by Mandelbrot [19]. The advancement of the computational tools further enriched the domain of the theory and analysis of fractals with diverse applications in almost all branches of sciences and engineering, for details on fractals, one may refer Barnsley [1-3], Edgar [9], Falconer [10], Kigami [17], Peitgen et al [21-22], Singh and Prasad [26] and several references thereof. Recently, Easwaramoorty and Uthayakumar [10] studied a Hutchinson-Barnsley (HB) operator on fuzzy metric spaces and presented an analysis on fractals in such spaces. Our aim is to extend and improve their results to multi fuzzy spaces.

The structure of the paper follows below. Section 2 provides the basic concepts and preliminaries required for the main results. In Section 3, by introducing multi iterated function system (MIFS) and multi fuzzy iterated function system (MFIFS) we define the notions of multi fuzzy fractal spaces in the sense of George and Veeramani [12]. Further, the results regarding the existence and uniqueness of attractors or fractals in such spaces are obtained. As an application, a collage theorem is derived in Section 4.

2. Preliminaries

Some basic concepts and definitions useful in the sequel are given first.

Definition 2.1 [2] *Let (X, d) be a complete metric space. Then the Hausdorff distance between points A and B in $\mathscr{K}(X)$, the collection of nonempty compact sub sets of X is defined by*

$$H_d(A, B) = \max\{d(A, B), d(B, A)\},$$

where $d(A, B) = \max\{\min\{d(x, y) \mid y \in B\} \mid x \in A\}$. *The Hausdorff space $(\mathscr{K}(X), H_d)$ is also called a Fractal space (see Barnsley [2]).*

Theorem 2.1 [2] *Let (X, d) be a metric space and $\mathscr{K}(X)$ be the collection of nonempty compact subsets of X with the Hausdorff metric H_d on $\mathscr{K}(X)$. Then $(\mathscr{K}(X), H_d)$ is complete iff (X, d) is complete.*

Definition 2.2 [2] *Suppose (X, d) is a complete metric space and $w_n : X \to X$ be contractions with the corresponding contractivity factors s_n, $n = 1, 2, 3, \ldots, N$. Then the system $\{X;\ w_n,\ n = 1, 2, 3, \ldots, N\}$ is called an iterated function system (IFS) in the metric space (X, d) with contractivity factor $s = \max\{s_n \mid n = 1, 2, 3, \ldots, N\}$.*

Theorem 2.2 [2, 26] *Let (X, d) be a complete metric space. Let $\{X;\ w_n, n = 1, 2, 3, \ldots, N\}$ be an IFS and W be the HB operator of the IFS. Then,*