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ORIGINAL ARTICLE

Implications on a Lattice

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Abstract In this paper, we study S-implication operator and QL-operation on a lattice L. We tabulate properties of S-implication and QL-operation with respect to different triangular norms and negations. We study properties of S-implication and QL-operation on uniquely complemented and orthocomplemented lattice.

Keywords Lattice · Implication · t-norm · t-conorm · Ideal

1. Introduction

Fuzzy implication operators were introduced to generalize classical implication operators. Bustince, Burillo and Soria [10] studied automorphism, negations and presented different implication operators. Baczynski and Jayaram [4] characterized (S,N)-implications on [0, 1]. Baczynski and Jayaram [5] made a survey of (S, N) and R-implications on [0, 1] and studied their properties. Cornelis, Deschrijver and Kerre [11] introduced implication operators in intuitionstic fuzzy set theory and interval valued fuzzy set theory. Reiser, Dimuro, Bedregal, Santos and Bedregal [27] defined S-implication on a complete lattice. Davvaz [13] obtained implication of fuzzy R-subgroup of a nearring with thresholds. Yang [29] studied fuzzy weak regular,

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strong pre-associative filters. Ajmal and Jahan [1] characterized normal L-subgroup of an L-group using an L-point. Fotea, Feng and Zhan [16] introduced fuzzy hypergroups. Mandal and Ranadive [26] obtained rough intuitionistic fuzzy substructures of fuzzy subring and studied their properties under homomorphism. Kedukodi, Kuncham and Bhavanari [21] introduced c-prime fuzzy ideals of nearrings. Kedukodi, Kuncham and Bhavanari [22], [23] studied equiprime, 3-prime and c-prime fuzzy ideals of nearrings and proposed reference point based rough set model. Tiwari, Sharan and Yadav [28] obtained relation between fuzzy rough sets, fuzzy closure spaces and fuzzy topology. Ajmal and Jahan [2] studied nilpotency of L-Subgroups of an L-Group. Bertoluzza, Doldi [7] studied distributivity between t-norms and t-conorms. Deschrijver [14] studied different representations of t-norms in interval valued fuzzy set theory. Jagadeesha, Kedukodi and Kuncham [19] introduced interval valued (i-v) L-fuzzy ideals on a lattice L by concurrently using a pair of t-norms and t-conorms. Kuncham, Kedukodi and Jagadeesha [25] studied properties of interval valued L-fuzzy ideals under homomorphism. Kedukodi, Jagadeesha and Kuncham [20] defined drastic t-norm and drastic t-conorm on a lattice using the properties of an ideal and a filter of a lattice. The drastic t-norm generalizes drastic product and drastic t-conorm generalizes drastic sum on [0, 1]. In this paper, we study S-implication and QL-operation on a complete bounded lattice and tabulate their properties with respect to different negations, t-norms and t-conorms. We study properties of S-implications on a uniquely complemented and orthocomplemented lattice.

2. Preliminaries

In this paper, $L =< L, \wedge_L, \vee_L >$ is a complete bounded lattice with a greatest element M and a least element m. $\leq_L$ denotes the partial order in L. We refer to Gratzer [17], Klement, Mesiar and Pap [24], Davvaz [12], Bhavanari and Kuncham [8], Bhavanari, Kuncham, Kedukodi [9] for more details on the topics involved in this paper.

Definition 2.1 [18] *A t-norm is a function $T : L \times L \to L$ such that $\forall\ x, y, z \in L$ following axioms are satisfied:*

1) *Commutativity: $T(x, y) = T(y, x)$,*

2) *Associativity: $T(x, T(y, z)) = T(T(x, y), z)$,*

3) *Monotonicity: If $y \leq_L z$, then $T(x, y) \leq_L T(x, z)$,*

4) *Boundary condition: $T(x, M) = x$.*

A t-norm T on L is called an idempotent t-norm if $T(x, x) = x\ \ \forall\ x \in L$.

Definition 2.2 [6] *A t-conorm is a function $C : L \times L \to L$, such that $\forall\ x, y, z \in L$ following axioms are satisfied:*

1) *Commutativity: $C(x, y) = C(y, x)$,*

2) *Associativity: $C(x, C(y, z)) = C(C(x, y), z)$,*

3) *Monotonicity: If $y \leq_L z$, then $C(x, y) \leq_L C(x, z)$,*