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Decomposition of Intuitionistic Fuzzy Matrices

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Abstract In this paper, we study some properties of modal operators in intuitionistic fuzzy matrix and we introduce a new composition operator and discuss some of its algebraic properties. Finally, we obtain a decomposition of an intuitionistic fuzzy matrix by using the new composition operator and modal operators.

Keywords Intuitionistic fuzzy matrix · Intuitionistic fuzzy set

1. Introduction

There have been theories evolved over the years to deal with the various types of uncertainties. These evolved theories are put into practice and when found to be wanting are improved upon, paving the way for new theories to handle the tricky uncertainties. The probability theory is one such important theory concerned with the analysis of random phenomena. In 1965, Zadeh [1] came out with the concept of fuzzy set which is indeed an extension of the classical notion of set. Fuzzy set has been found

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to be an effective tool to deal with fuzziness. However, it often falls short of the expected standard when describing the neutral state. As a result, a new concept namely intuitionistic fuzzy set (IFS) was worked out and the same was introduced in 1983 by Atanassov [2, 3]. Using the concept of IFS, Im et.al [4] studied intuitionistic fuzzy matrix (IFM).

IFM generalizes the fuzzy matrix introduced by Thomson [5] and has been useful in dealing with areas such as decision making, relational equations, clustering analysis etc. A number of authors [6, 7] have effectively presented impressive results using fuzzy matrix. Atanassov [8], using the definition of index matrix, has paved way for intuitionistic fuzzy index matrix and has further extending it to temporal intuitionistic fuzzy index matrix. IFM is also very useful in the discussion of intuitionistic fuzzy relation [9, 10]. Z.S. Xu [11, 12] studied intuitionistic fuzzy value and also IFMs. He defined intuitionistic fuzzy similarity relation and also utilized it in clustering analysis.

A lot of research activities have been carried out over the years on IFMs in [13-17]. The period of powers of square IFMs is discussed at length along with some of the results for the equivalence IFM by Jeong and Park [18] while Pal et al. [19-26] made a comprehensive study and neatly developed IFM in various years. Another researcher namely Mondal [27] attempted a study on the similarity relations, invertibility and eigenvalues of IFM. In [28], a research was carried out on how a transitive IFM decomposed into a sum of nilpotent IFM and symmetric IFM and in [29] how an IFM gets decomposed into a product of idempotent IFM and rectangular IFM.

Atanassov introduced modal operators in [2] which are meaningless in fuzzy set theory and found a promising direction in research. The above operators for IFMs and some results are obtained in [30]. In this paper, some necessary and sufficient conditions are discussed for a transitive and c-transitive closure matrix interms of modal operators. we explore some more results using modal operators for IFM under max-min composition and discuss similarity relation, idempotents etc. Finally, using modal operators we decompose an IFM by introducing a new composition operator and some properties of that new operator are proved.

2. Preliminaries

We recollect some relevant basic definitions and results will be used later.

Definition 2.1 *Let a set* $X = \{x_1, x_2, \cdots x_n\}$ *be fixed. Then an IFS* [2] *can be defined as* $A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle \mid x_i \in X\}$ *which assigns to each element* x_i *a membership degree* $\mu_A(x_i)$ *and a non membership degree* $\nu_A(x_i)$*, with the condition* $0 \le \mu_A(x_i) + \nu_A(x_i) \le 1$ *for every* $x_i \in X$.

Definition 2.2 *Xu and Yager called the* 2*-tuple* $\alpha(x_i) = (\mu_\alpha(x_i), \nu_\alpha(x_i))$ *an intuitionistic fuzzy value (IFV)* [11, 12] *where* $\mu_\alpha(x_i) \in [0,1]$*,* $\nu_\alpha(x_i)) \in [0,1]$ *and* $\mu_\alpha(x_i) + \nu_\alpha(x_i) \le 1$.

Definition 2.3 [2] *Let* $\langle x, x'\rangle, \langle y, y'\rangle \in IFS$*. Then we have*

(i) $\langle x, x'\rangle \vee \langle y, y'\rangle = \langle \max\{x, y\}, \min\{x', y'\}\rangle$.

(ii) $\langle x, x'\rangle \wedge \langle y, y'\rangle = \langle \min\{x, y\}, \max\{x', y'\}\rangle$.