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Selling two goods optimally ☆,☆☆

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We provide sufficient conditions for revenue maximization in a two-good monopoly where the buyer's values for the items come from independent (but not necessarily identical) distributions over bounded intervals. Under certain distributional assumptions, we give exact, closed-form formulas for the prices and allocation rule of the optimal selling mechanism. As a side result we give the first example of an optimal mechanism in an i.i.d. setting over a support of the form $[0, b]$ which is *not* deterministic. Since our framework is based on duality techniques, we were also able to demonstrate how slightly relaxed versions of it can still be used to design mechanisms that have very good approximation ratios with respect to the optimal revenue, through a "convexification" process.

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1. Introduction

The problem of designing auctions that maximize the seller's revenue in settings with many heterogeneous goods has attracted a large amount of interest in the last years, both from the Computer Science as well as the Economics community (see e.g. [2–10]). Here the seller faces a buyer whose true values for the m items come from a probability distribution over $\mathbb{R}_+^m$ and, based only on this incomplete prior knowledge, he wishes to design a selling mechanism that will maximize his expected revenue. For the purposes of this paper, the prior distribution is a product one, meaning that the item values are independent. The buyer is additive, in the sense that her happiness from receiving any subset of items is the sum of her values of the individual items in that bundle. The buyer is also selfish and completely rational, thus willing to lie about her true values if this is to improve her own happiness. So, the seller should also make sure to give the right incentives to the buyer in order to avoid manipulation of the protocol by misreporting.

The special case of a single item has been very well understood since the seminal work of Myerson [11]. However, when one moves to settings with multiple goods, the problem becomes notoriously difficult and novel approaches are necessary. Despite the significant effort of the researchers in the field, essentially only specialized, partial results are known: there are exact solutions for two items in the case of identical uniform distributions over unit-length intervals [2,3], exponential over $[0, \infty)$ [7] or identical Pareto distributions with tail index parameters $\alpha \geq 1/2$ [4]. For more than two items, optimal results are only known for uniform values over the unit interval [8], and due to the difficulty of exact solutions most of the work focuses in showing approximation guarantees for simple selling mechanisms [4,12–16]. This difficulty is further supported

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by the complexity (#P-hardness) results of Daskalakis et al. [6]. It is important to point out that even for two items *we know of no general and simple, closed-form conditions framework under which optimality can be extracted when given as input the item distributions, in the case when these are not necessarily identical.* This is our goal in the current paper.

Our contribution We introduce general but simple and clear, closed-form distributional conditions that can guarantee optimality and immediately give the form of the revenue-maximizing selling mechanism (its payment and allocation rules), for the setting of two goods with values distributed over bounded intervals (Theorem 1). For simplicity and a clearer exposition we study distributions supported over the real unit interval $[0, 1]$. By scaling, the results generalize immediately to intervals that start at 0, but more work would be needed to generalize them to arbitrary intervals. We use the closed forms to get optimal solutions for a wide class of distributions satisfying certain simple analytic assumptions (Theorem 2 and Sect. 4). As useful examples, we provide exact solutions for families of monomial ($\propto x^c$) and exponential ($\propto e^{-\lambda x}$) distributions (Corollaries 1 and 2 and Sect. 4), and also near-optimal results for power-law ($\propto (x+1)^{-\alpha}$) distributions (Sect. 5). This last approximation is an application of a more general result (Theorem 3) involving the relaxation of some of the conditions for optimality in the main Theorem 1; the "solution" one gets in this new setting might not always correspond to a feasible selling mechanism, however it still provides an upper bound on the optimal revenue as well as hints as to how to design a well-performing mechanism, by "convexifying" it into a feasible mechanism (Sect. 5).

Particularly for the family of monomial distributions it turns out that the optimal mechanism is a very simple deterministic mechanism that offers to the seller a menu of size just 4 (using the menu-complexity notion of Hart and Nisan [17,18]): fixed prices for each one of the two items and for their bundle, as well as the option of not buying any of them. For other distributions studied in the current paper randomization is essential for optimality, as is generally expected in such problems of multidimensional revenue maximization (see e.g. [3,5,7]). For example, this is the case for two i.i.d. exponential distributions over the unit interval $[0, 1]$, which gives the first such example where determinism is suboptimal even for regularly[1] i.i.d. items. A point worth noting here is the striking difference between this result and previous results [7, 14] about i.i.d. exponential distributions which have as support the entire $\mathbb{R}_+$: the optimal selling mechanism there is the deterministic one that just offers the full bundle of both items.

Although the conditions that the probability distributions must satisfy are quite general, they leave out a large class of distributions. For example, they do not apply to power-law distributions with parameter $\alpha > 2$. In other words, this work goes some way towards the complete solution for arbitrary distributions for two items, but the general problem is still open. In this paper, we opted towards simple conditions rather than full generality, but we believe that extensions of our method can generalize significantly the range of distributions; we expect that a proper "ironing" procedure will enable our technique to resolve the general problem for two items.

Techniques The main result of the paper (Theorem 1) is proven by utilizing the *duality* framework of [8] for revenue maximization, and in particular using complementarity: the optimality of the proposed selling mechanism is shown by verifying the existence of a dual solution with which they satisfy together the required complementary slackness conditions of the duality formulation. Constructing these dual solutions explicitly seems to be a very challenging task and in fact there might not even be a concise way to do it, especially in closed-form. So instead we just prove the existence of such a dual solution, using a *max-flow min-cut* argument as main tool (Lemma 3, Fig. 2). This is, in a way, an abstraction of a technique followed in [8] for the case of uniform distributions which was based on Hall's theorem for bipartite matchings. Since here we are dealing with general and non-identical distributions, this kind of refinement is essential and non-trivial, and in fact forms the most technical part of the paper. Our approach has a strong geometric flavour, enabled by introducing the notion of the *deficiency* of a two-dimensional body (Definition 1, Lemma 2), which is inspired by classic matching theory [19,20].

1.1. Model and notation

We study a two-good monopoly setting in which a seller deals with a buyer who has values $x_1, x_2 \in I$ for the items, where $I = [0, 1]$. The seller has only an incomplete knowledge of the buyer's preference, in the form of two independent distributions (with densities) f_1, f_2 over I from which x_1 and x_2 are drawn, respectively. The cdf of f_j will be denoted by F_j. As in the seminal work of Myerson [11], the density functions will be assumed to be absolutely continuous and positive. We will also use vector notation $\mathbf{x} = (x_1, x_2)$. For any item $j \in \{1, 2\}$, index $-j$ will refer the complementary item, that is $3 - j$, and as it's standard in game theory $\mathbf{x}_{-j} = x_{-j}$ will denote the remaining of vector $\mathbf{x}$ if the j-th coordinate is removed, so $\mathbf{x} = (x_j, x_{-j})$ for any $j = 1, 2$.

The seller's goal is to design a selling mechanism that will maximize his revenue. Without loss[2] we can focus on direct-revelation mechanisms: the bidder will be asked to submit bids b_1, b_2 and the mechanism consists simply of an allocation rule $a_1, a_2 : I^2 \to I$ and a payment function $p : I^2 \to \mathbb{R}_+$ such that $a_j(b_1, b_2)$ is the probability of item j being sold to the buyer (notice how we allow for randomized mechanisms, i.e. lotteries) and $p(b_1, b_2)$ is the payment that the buyer expects to pay; it is easier to consider the expected payment for all allocations, rather than individual payments that depend on

[1] A probability distribution F is called *regular* if $t - \frac{1-F(t)}{f(t)}$ is increasing. This quantity is known as the *virtual valuation*.

[2] This is due to the celebrated Revelation Principle [11].

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