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Checking dynamic consistency of conditional hyper temporal networks via mean payoff games
Hardness and (pseudo) singly-exponential time algorithm

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ABSTRACT

Conditional Simple Temporal Network (CSTN) is a constraint-based graph formalism for conditional temporal planning, which may be viewed as an extension of Simple Temporal Networks. Recently, STNs have been generalized into Hyper Temporal Networks (HyTNs), by considering weighted directed hypergraphs where each hyperarc models a disjunctive temporal constraint. We introduce the *Conditional Hyper Temporal Network (CHyTN)* model, a natural extension and generalization of both CSTNs and HyTNs, obtained by blending them together. We show that deciding whether a given CSTN is dynamically-consistent is coNP-hard, and that deciding whether a given CHyTN is dynamically-consistent is PSPACE-hard. Next, we offer the first deterministic (pseudo) singly-exponential time algorithm for checking DC in CHyTNs and CSTNs. To analyze the computational complexity of the proposed algorithm, we introduce a refined notion of DC, named ϵ-*DC*, presenting a sharp lower bounding analysis on the critical value of the reaction time where a conditional temporal network transits from being, to not being, dynamically-consistent.

1. Introduction and motivation

In many areas of Artificial Intelligence (AI), including temporal planning and scheduling, the representation and management of quantitative temporal aspects is of crucial importance (see e.g., [26,27,17,3,10,9]). Examples of possible quantitative temporal aspects include constraints on the earliest start time and latest end time of activities and constraints over the minimum and maximum temporal distance between activities. In many cases these constraints can be represented by *Simple Temporal Networks* (STNs) [16], i.e., directed weighted graphs where nodes represent events to be scheduled in time and arcs represent temporal distance constraints between pairs of events. Recently, STNs have been generalized into *Hyper Temporal Networks* (HyTNs) [12,13], a strict generalization of STNs introduced to overcome the limitation of considering only conjunctions of constraints, but maintaining a practical efficiency in the consistency checking of the instances. In a HyTN a single temporal hyperarc constraint is defined as a set of two or more maximum delay constraints which is satisfied when at least one of these delay constraints is satisfied. HyTNs are meant as a light generalization of STNs offering an interesting compromise. On one side, there exist practical pseudo-polynomial time algorithms for checking the consistency of HyTNs and

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computing feasible schedules for them. On the other side, HyTNs offer a more powerful model accommodating natural disjunctive constraints that cannot be expressed by STNs. In particular, HyTNs are weighted directed hypergraphs where each hyperarc models a disjunctive temporal constraint called *hyperconstraint*. The computational equivalence between checking consistency in HyTNs and determining winning regions in *Mean Payoff Games* (MPGs) [18,30,5] was also pointed out in [12,13], where the approach was shown to be robust thanks to experimental evaluations (also see [4]). MPGs are a family of 2-player infinite pebble games played on finite graphs which is well known for having theoretical interest in computational complexity, being one of the few natural problems lying in $\mathsf{NP} \cap \mathsf{coNP}$, as well as various applications in model checking and formal verification [20].

However, in the representation of quantitative temporal aspects of systems, *conditional* temporal constraints pose a serious challenge for conditional temporal planning, where a planning agent has to determine whether a candidate plan will satisfy the specified conditional temporal constraints. This can be difficult because the temporal assignments that satisfy the constraints associated with one conditional branch may fail to satisfy the constraints along a different branch (see, e.g., [29]). The present work unveils that HyTNs and MPGs are a natural underlying combinatorial model for checking the consistency of certain conditional temporal problems that are known in the literature and that are useful in some practical applications of temporal planning, especially, for managing the temporal aspects of Workflow Management Systems (WfMSs) [3,10] and for modeling Healthcare's Clinical Pathways [9]. Thus we focus on *Conditional Simple Temporal Networks (CSTNs)* [29,22], a constraint-based model for conditional temporal planning. The CSTN formalism extends STNs in that: (1) some of the nodes are *observation events*, to each of them is associated a boolean variable whose value is disclosed only at execution time; (2) *labels* (i.e. conjunctions over the literals) are attached to all nodes *and* constraints, to indicate the situations in which each of them is required. The planning agent (or *Planner*) must schedule all the required nodes, meanwhile respecting all the required temporal constraints among them. This extended framework allows for the off-line construction of conditional plans that are guaranteed to satisfy complex networks of temporal constraints. Importantly, this can be achieved even while allowing for the decisions about the precise timing of actions to be postponed until execution time, in a least-commitment manner, thereby adding flexibility and making it possible to adapt the plan dynamically, in response to the observations that are made during execution (see [29] for further details and examples).

Three notions of consistency arise for CSTNs: weak, strong, and *dynamic*. Dynamic consistency (DC) is the most interesting one; it requires the existence of conditional plans where decisions about the precise timing of actions are postponed until execution time, but it nonetheless guarantees that all the relevant constraints will be ultimately satisfied. Still, it is the most challenging and it was conjectured to be hard to assess [29]. Indeed, to the best of our knowledge, the tightest currently known upper bound on the time complexity of deciding whether a given CSTN is dynamically-consistent is doubly-exponential time [29]. It first builds an equivalent Disjunctive Temporal Problem (DTP) of size exponential in the input CSTN, and then applies to it an exponential-time DTP solver to check its consistency. However, this approach turns out to be quite limited in practice: experimental studies have already shown that the resolution procedures, as well as the currently known heuristics, for solving general DTPs become quite burdensome with 30 to 35 DTP variables (see e.g., [28,24,25]), thus dampening the practical applicability of the approach.

1.1. Contribution

In this work we introduce and study the *Conditional Hyper Temporal Network (CHyTN)* model, a natural extension and generalization of both the CSTN and the HyTN model which is obtained by blending them together. One motivation for studying it is to transpose some benefits and opportunities for application, that have arisen from the introduction of HyTNs (see [12,13]), to the context of *conditional* temporal planning. In so doing our main contribution is that to offer the first sound-and-complete deterministic (pseudo) singly-exponential time algorithm for checking the dynamic consistency of CSTNs. Soon after a formal introduction to the models, we show that deciding whether a given CSTN is dynamically-consistent is coNP-hard. Then, we offer a proof that deciding whether a given CHyTN is dynamically-consistent is PSPACE-hard, provided that the input instances are allowed to include both multi-head and multi-tail hyperarcs. In light of this, we focus on CHyTNs that allow only multi-head hyperarcs. Concerning multi-head CHyTNs, perhaps most importantly, we unveil a connection between the problem of checking their dynamic consistency and that of determining winning regions in MPGs (of a singly-exponential size in the number of propositional variables of the input CHyTN), thus providing the first sound-and-complete (pseudo) singly-exponential time algorithm for this same task of deciding the dynamic consistency and yielding a dynamic execution strategy for multi-head CHyTNs. The resulting worst-case time complexity of the DC-Checking procedure is $O\big(2^{3|P|}|V||\mathcal{A}|m_{\mathcal{A}} + 2^{4|P|}|V|^2|\mathcal{A}||P| + 2^{4|P|}|V|^2 m_{\mathcal{A}} + 2^{5|P|}|V|^3|P|\big)W$, where $|P|$ is the number of propositional variables, $|V|$ is the number of event nodes, $|\mathcal{A}|$ is the number of hyperarcs, $m_{\mathcal{A}}$ is the size (i.e., roughly, the encoding length of $\mathcal{A}$), and W is the maximum absolute integer value of the weights of the input CHyTN. The algorithm is still based on representing a given instance on an exponentially sized network, as first suggested in [29]. The difference, however, is that we propose to map CSTNs and CHyTNs on (exponentially sized) HyTNs/MPGs rather than on DTPs. This makes an important difference, because the consistency check for HyTNs can be reduced to determining winning regions in MPGs, as shown in [12,13], which admits practical and effective pseudo-polynomial time algorithms (in some special cases the algorithms for determining winning regions in MPGs exhibit even a strongly polynomial time behavior, see e.g., [13,4,1,7]). To summarize, we obtain an improved upper bound on the theoretical time complexity of the DC-checking of CSTNs (i.e., from 2-EXP to pseudo-$E \cap \mathsf{NE} \cap \mathsf{coNE}$) together with a faster DC-checking procedure, which can be used on CHyTNs with a larger num-