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Descriptional complexity of limited automata

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ABSTRACT

A k-limited automaton is a linear bounded automaton that may rewrite each tape cell only in the first k visits, where $k \geq 0$ is a fixed constant. It is known that these automata accept context-free languages only. We investigate the descriptional complexity of limited automata. Since the unary languages accepted are necessarily regular, we first study the cost in the number of states when finite automata simulate a unary k-limited automaton. For the conversion of a $4n$-state deterministic 1-limited automaton into one-way or two-way deterministic or nondeterministic finite automata, we show a lower bound of $n \cdot F(n)$ states, where F denotes Landau's function. So, even the ability to deterministically rewrite any cell only once gives an enormous descriptional power. For the simulation cost for removing the ability to rewrite each cell $k \geq 1$ times, more precisely, the cost for the simulation of sweeping unary k-limited automata by deterministic finite automata, we obtain a lower bound of $n \cdot F(n)^k$. The upper bound of the cost for the simulation by two-way deterministic finite automata is a polynomial whose degree is quadratic in k. If the k-limited automaton is rotating, the upper bound reduces to $O(n^{k+1})$ and the lower bound derived is $\Omega(n^{k+1})$ even for nondeterministic two-way finite automata. So, for rotating k-limited automata, the trade-off for the simulation is tight in the order of magnitude. Finally, we consider the simulation of k-limited automata over general alphabets by pushdown automata. It turns out that the cost is an exponential blow-up of the size. Furthermore, an exponential size is also necessary.

1. Introduction

The cost for the simulation of one formal model by another is one of the main topics of descriptional complexity, where the cost is measured in close connection to the sizes of the models. Such simulations are of particular interest when both formal models capture the same family of languages. A fundamental result is that nondeterministic finite automata can be simulated by deterministic finite automata by paying the cost of exponentially many states (see, for example, [16]). Among the many models characterizing the regular languages, an interesting variant is the linear bounded automata where the time is restricted as well. It was shown by Hennie [5] that linear-time computations cannot accept non-regular languages. This result has been improved to $o(n \log n)$ time by Hartmanis [4]. In particular, the former result implies that a linear bounded automaton where any tape cell may be visited only a constant number of times accepts a regular language. Recent results [24] showed that the upper as well as the lower bound for converting a weight-reducing machine of this type, that

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is, each transition is required to lower the weight of the scanned symbol, into a deterministic finite automaton is doubly exponential. A related result [1] showed that if a two-way finite automaton is allowed to freely place a pebble on the tape, then again no non-regular language can be accepted, even if the time is unlimited. Doubly exponential upper and a lower bounds for the simulation by a deterministic finite automaton have been derived [16].

A variant of the machines studied by Hennie [5] was introduced by Hibbard [6]. He investigated linear bounded automata that may rewrite each tape cell only in the first k visits, where k is a fixed constant. However, afterwards the cells can still be visited any number of times (but without rewriting their contents). Hibbard [6] showed that the nondeterministic variant characterizes the context-free languages provided $k \geq 2$, while there is a tight and strict hierarchy of language classes depending on k for the deterministic variant. The latter means that the family of languages accepted with k rewrites is strictly included in the family of languages accepted with $k+1$ rewrites. One-limited automata, deterministic and nondeterministic, can accept only regular languages. From these results it follows that any *unary* k-limited automaton accepts a regular language only.

Recently, the study of limited automata from the descriptional complexity point of view has been initiated by Pighizzini and Pisoni [20,21]. In [21] it was shown that the deterministic 2-limited automata characterize the deterministic context-free languages, which complements the result on nondeterministic machines. Furthermore, conversions between 2-limited automata and pushdown automata have been investigated. For the deterministic case the upper bound for the conversion from 2-limited automata to pushdown automata is doubly exponential. Conversely, the trade-off is shown to be polynomial. If the automata are nondeterministic, exponential upper and lower bounds are derived for the 2-limited automata to pushdown automata conversion. Comparisons between 1-limited automata and finite automata were done in [20]. In particular, a double exponential trade-off between nondeterministic 1-limited automata and one-way deterministic finite automata was shown. For deterministic 1-limited automata the conversion cost a single exponential increase in size. These results imply an exponential trade-off between nondeterministic and deterministic 1-limited automata, and they show that 1-limited automata can have less states than equivalent two-way nondeterministic finite automata.

For a restricted variant of limited automata, so-called strongly limited automata, it was shown that context-free grammars as well as pushdown automata can be transformed in strongly limited automata and vice versa with polynomial cost [19].

Here, we first consider deterministic k-limited automata accepting unary languages. The descriptional complexity of unary regular languages has been studied in many ways. On one hand, many automata models such as one-way finite automata, two-way finite automata, pushdown automata, or context-free grammars for unary languages were investigated and compared to each other with respect to simulation results and the size of the simulation (see, for example, [3,15,18,23]). On the other hand, many results concerning the state complexity of operations on unary languages have been obtained (see, for example, [7,10,14,22]).

The results on the expressive power of limited automata imply that any unary language accepted by some k-limited automaton is regular. So, it is of interest to investigate the descriptional complexity in comparison with the models mentioned above. We establish upper and lower bounds for the conversion of unary deterministic k-limited automata to one-way and two-way finite automata. Moreover, the simulation of general k-limited automata by pushdown automata is considered. It turns out that the cost is an exponential blow-up of the size. From the case of 2-limited automata [21], it turns out that an exponential gap is also necessary.

2. Preliminaries

We write Σ^* for the set of all words over the finite alphabet Σ. The empty word is denoted by λ, the reversal of a word w by w^R, and for the length of w we write $|w|$. We use $\subseteq$ for inclusions and $\subset$ for strict inclusions.

Let $k \geq 0$ be an integer. A k-limited automaton is a restricted linear bounded automaton. It consists of a finite state control and a read–write tape whose initial content is the input word in between two endmarkers. At the outset of a computation, the automaton is in the designated initial state and the head of the tape scans the left endmarker. Depending on the current state and the currently scanned symbol on the tape, the automaton changes its state, rewrites the current symbol on the tape, and moves the head one cell to the left or one cell to the right. However, the rewriting is restricted such that the machine may rewrite each tape cell only in the first k visits. Subsequently, the cell can still be scanned but the content cannot be changed any longer. So, a 0-limited automaton is a two-way finite automaton. An input is accepted if the machine reaches an accepting state and halts.

The original definition of such devices by Hibbard [6] is based on string rewriting systems whose sentential forms are seen as configurations of automata. Let $u_1u_2\cdots u_{i-1}su_iu_{i+1}\cdots u_n$ be a sentential form that represents the tape contents $u_1u_2\cdots u_n$ and the current state s. Basically, in [6] rewriting rules were provided of the form $su_i \to u_i's'$, which means that the state changes from s to s', the tape cell to the right of s is scanned and rewritten from u_i to u_i', the input head is moved to the right, and $u_{i-1}s \to s'u_{i-1}'$, which means that the state changes from s to s', the tape cell to the left of s is scanned and rewritten from u_{i-1} to u_{i-1}', the input head is moved to the left. In this context, an automaton that changes its head direction on a cell scans the cell twice. By Pighizzini and Pisoni [20,21] and below, limited automata are defined in a way that reflects this behavior.

Formally, a *(nondeterministic) k-limited automaton* (k-LA, for short) is a system $M = \langle S, \Sigma, \Gamma, \delta, \rhd, \lhd, s_0, F\rangle$, where S is the finite, nonempty set of *internal states*, Σ is the finite set of *input symbols*, Γ is the finite set of *tape symbols* partitioned