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Handling infinitely branching well-structured transition systems ☆

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ABSTRACT

Most decidability results concerning well-structured transition systems apply to the *finitely branching* variant. Yet some models (inserting automata, ω-Petri nets, ...) are naturally infinitely branching. Here we develop tools to handle infinitely branching WSTS by exploiting the crucial property that in the (ideal) completion of a well-quasi-ordered set, downward-closed sets are *finite* unions of ideals. Then, using these tools, we derive decidability results and we delineate the undecidability frontier in the case of the termination, the maintainability and the coverability problems. Coverability and boundedness under new effectiveness conditions are shown decidable.

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1. Introduction

Well-structured transition systems (WSTS) [1–3] as a general class of infinite-state systems have spawned decidability results for important problems such as termination, boundedness, maintainability and coverability. WSTS consist of a (usually infinite) well ordered set of states, together with a monotone transition relation. WSTS have found multiple uses: in settling the decidability status of reachability and coverability for graph transformation systems [4,5], in the forward analysis of depth-bounded processes [6,7], in the verification of parameterized protocols [8] and the verification of multi-threaded asynchronous software [9]. WSTS remain under development and are actively being investigated [10–15].

Most existing decidability results for WSTS apply to the *finitely branching* variant. However, there are many intrinsically *infinitely branching* WSTS. Let us cite inserting FIFO automata [16] which are able to insert any word at any time in a FIFO buffer, inserting automata [17], recursive-parallel systems [18] and ω-Petri nets [19]. Moreover, any finitely branching WSTS parameterized with an infinite set of initial states (such as broadcast protocols [8]) also inherits an infinitely branching state. For instance, Geeraerts, Heußner, Praveen and Raskin argue in [19] that parametric concurrent systems with dynamic thread

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creation can naturally be modelled by some classes of infinitely branching systems, like ω-Petri nets, *i.e.* Petri net with arcs that can consume/create arbitrarily many tokens.

The primary motivation for this paper is to explore the decidability status of the termination, boundedness, maintainability and coverability problems for infinitely branching (general) WSTS. For the coverability problem, known to be decidable for WSTS fulfilling upward pre-effectiveness [3] (which roughly means computability of a finite basis of the upward closure of the set of immediate predecessors, the testing of which is provably undecidable in some WSTS), we wish to draw from the recent algebra-theoretic characterizations of downward-closed sets [10] and conceive of a post-oriented computability hypothesis suitable for the design of a forward algorithm. Indeed, forward algorithms are arguably more intuitive than backward algorithms and post-oriented computability more easily verified than pre-oriented computability. Our contributions are the following:

1. As technical tools, we simplify and extend the analysis of the completion of a general WSTS and we relate the behaviour of a WSTS to that of its completion. In particular, we provide a general presentation of the completion that is much less daunting than the presentations currently available in the literature. This sets the stage for exploiting the main property of the completion of a WSTS, namely, the expressibility of any downward-closed set as a (unique, as shown here) finite union of ideals, in the design of algorithms.
2. We uncover a new termination property (called *strong* termination) that is computationally equivalent to the usual termination property for finitely branching WSTS but that subtly differs from it in the presence of infinitely branching WSTS. Indeed, we exhibit WSTS for which strong termination is decidable yet the usual termination is undecidable. A similar subtle issue arises as well in our generalization of the maintainability problem to infinitely branching.
3. We generalize most decidability results mentioned for finitely branching WSTS earlier to the infinitely branching case. This requires carefully tracking the effectiveness and the monotonicity conditions which support decidability. When possible, we delineate the frontier between decidability for a problem and the undecidability that results from dropping one of these conditions. The new decidability results for (strong) termination and (weak) maintainability exploit the completion. An outcome of our work is that the finite tree construction technique can be recovered, even in the infinitely branching case, for the purpose of deciding the boundedness problem for example. The new algorithm for coverability uses a forward strategy coupled with a post-oriented computability hypothesis.

Section 2 below fixes the notation pertaining to orderings and transition systems. Section 3 recalls the notion of WSTS, gives examples, discusses branching and effectiveness, defines the computation problems at issue and adds two undecidability results concerning finitely branching WSTS. Section 4 develops tools to handle infinitely branching WSTS and forms the theoretical backbone of our paper. Section 5 contains our decidability results for infinitely branching WSTS. Section 6 summarizes and suggests future work.

2. Preliminaries

2.1. Orderings

Let X be a set and $\leq \subseteq X \times X$. We say that $\leq$ is a *quasi-ordering* (*qo*) for X if it is reflexive and transitive. If $\leq$ is also antisymmetric, then it is a *partial ordering* (*po*). A quasi-ordering (resp. partial ordering) $\leq$ is said to be a *well-quasi-ordering* (resp. *well partial ordering*), abbreviated *wqo* (resp. *wpo*), if for every infinite sequence $x_0, x_1, \ldots$ of elements $x_n \in X$, there exist $i < j$ such that $x_i \leq x_j$.

It is well-known that $\mathbb{N}^d$ is well partially ordered under $\leq_{\mathbb{N}^d}$ defined by

$$(x_1, x_2, \ldots, x_d) \leq_{\mathbb{N}^d} (x'_1, x'_2, \ldots, x'_d) \iff \forall i \in \{1, 2, \ldots, d\}\ x_i \leq x'_i .$$

In this work, we extend $\mathbb{N}$ to $\mathbb{N}_\omega \stackrel{\text{def}}{=} \mathbb{N} \cup \{\omega\}$ and we extend $\leq_{\mathbb{N}}$ to $\leq_{\mathbb{N}_\omega}$ with $x \leq_{\mathbb{N}_\omega} \omega$ for all $x \in \mathbb{N}_\omega$. The quasi-ordering $\leq_{\mathbb{N}_\omega}$ is also a wpo and is naturally extended to the wpo $\leq_{\mathbb{N}^d_\omega}$ over $\mathbb{N}^d_\omega$. We will simply write $\leq$ for $\leq_{\mathbb{N}}$, $\leq_{\mathbb{N}_\omega}$, $\leq_{\mathbb{N}^d}$ and $\leq_{\mathbb{N}^d_\omega}$ when there is no ambiguity. We also write $x < y$ whenever $x \leq y$ and $\neg(y \leq x)$. In some examples, we will also consider the subword ordering denoted $\preceq$. For every finite alphabet Σ and $u, v \in \Sigma^*$, $u \preceq v$ if, and only if, $u = v$ or u can be obtained from v by removing some letters. It is well-known that $\preceq$ is a wqo.

Let $T \subseteq X$. We define the *upward closure* of T as $\uparrow T \stackrel{\text{def}}{=} \{x \in X : y \leq x \text{ for some } y \in T\}$ and the *downward closure* of T as $\downarrow T \stackrel{\text{def}}{=} \{x \in X : x \leq y \text{ for some } y \in T\}$. We say that T is *upward closed* if $T = \uparrow T$ and *downward closed* if $T = \downarrow T$. Let $x \in X$, we simply write $\uparrow x$ for $\uparrow \{x\}$, and $\downarrow x$ for $\downarrow \{x\}$. An *(upward) basis* of an upward closed set T is a set B such that $T = \uparrow B$. It is known that every upward closed subset of a well-quasi-ordered set has a minimal finite basis. An *ideal* I is a downward closed subset of X that is also *directed*, *i.e.*, nonempty and such that $\forall a, b \in I$, $\exists c \in I$ such that $a \leq c$ and $b \leq c$. We define $\mathsf{Ideals}(X)$ as the set of ideals of X, *i.e.*, $\mathsf{Ideals}(X) \stackrel{\text{def}}{=} \{\emptyset \subset I \subseteq X : I = \downarrow I \text{ and } I \text{ is directed}\}$.

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