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Minimizing the number of max-power users in ad-hoc wireless networks with minimum node degree requirements

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ABSTRACT

We consider the problem of assigning one of two possible transmission ranges to each node of a wireless ad-hoc network, aiming for a predefined degree of redundancy and reliability by requiring that each node has at least $\Delta \in \mathbb{N}$ neighbors in the resulting network. To conserve the networks limited energy, this goal should be achieved with as few as possible nodes using the higher of the two transmission ranges. We show that this so-called Two Power Level Vertex Degree problem in NP-complete, derive an approximation algorithm that utilizes an arbitrary approximation algorithm for the well-known Min Set Multicover problem and show that the approximation quality of this algorithm transfers to our algorithm.

1. Introduction

A sensor network consists of small devices, so-called sensor nodes, which are deployed across an area and communicate with each other through a wireless radio channel while monitoring certain aspects of the environment. Since they run on a limited battery, it is essential to reduce their power consumption to maximize the network's lifetime. A common practice is to adjust the transmission power that each nodes uses under the constraint that the resulting network topology is connected. This class of problems is commonly known as Range Assignment (RA) problems.

There are several studies on RA problems with different preliminaries, for example [4,8]. The RA problem is typically defined as computing a range assignment $f : P \to \mathbb{R}^+$ for a set of points $P \subset \mathbb{R}^n$ $(1 \le n \le 3)$, representing the nodes in the network, such that the total energy $\sum_{p \in P} c(f(p))$ is minimal (c being a cost function according to a radio wave propagation model) under the constraint that the graph (P, E), $E := \{(p,q) \in P^2 \mid \|p-q\|_2 \le f(p)\}$ is strongly connected, where $\|p-q\|_2$ denotes the euclidean distance between p and q. It is well known that the corresponding decision problem is NP-hard for both the 2- and 3-dimensional euclidean space [9,5]. Variations of the problem include requiring connectivity of the resulting undirected graph (where an edge $\{u,v\}$ is present if and only if both directed edges (u,v) and (v,u) are present) instead of strong connectivity [1,2] and restricting the possible power level values from real numbers to a discrete space [6] or limiting the number of power levels to two [3].

In this paper we examine a sensor network with devices that are able to choose between a low and a high transmission power level and consider a different topological aspect of the network that relates to redundancy and fault tolerance: A desired minimum degree (i.e. a minimum number of neighbors) at each sensor node. Since the nodes are only able to choose between two different power levels, the task of minimizing the overall power consumption is simplified to determining a minimum cardinality subset of nodes that utilize the higher

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transmission power. We present and analyze an algorithm for this problems that inherits the approximation quality of an arbitrary approximation algorithm for the well-known MIN SET MULTICOVER problem. We also consider the additional restriction of the network topology obeying the commonly used unit disk graph model and show that in this case the problems can be approximated with an improved quality. Finally, we also show the NP-completeness of the decision problem that corresponds to this range assignment problem, i.e. the task of deciding whether it is possible to compute a solution that consists of at most k nodes for an additionally given positive integer k.

In section 2 we will introduce basic definition and formally define the range assignment problem that we examine before presenting an approximation algorithm in section 3. Finally, in section 4 we prove the NP-completeness of the considered problem.

2. Definitions and terminology

A pair (V, E) is an *undirected graph* with *vertex set* V and *edge set* E, if V is a finite set and $E \subseteq \{\{u, v\} \mid u, v \in V, u \neq v\}$. With respect to networks, the terms *vertex* and *node* are used interchangeably.

For an undirected graph $G = (V, E)$, an edge $e \in E$ is *incident* to a vertex $v \in V$, if $v \in e$ and two vertices $u, v \in V$ are *adjacent*, if $\{u, v\} \in E$. The *degree* of v in G, denoted by $\delta_G(v)$, is the number of edges incident to v.

The networks we consider are modeled as an undirected graph (V, E), which is the network topology for the case that all nodes use minimal transmission power, and a mapping from V to the power set $\mathcal{P}(V)$ of V that describes the set of nodes $r(v)$ a node $v \in V$ can additionally reach by using increased transmission power.

Definition 1. *Let $G = (V, E)$ be an undirected graph, $U \subseteq V$ a subset of vertices and $r : V \mapsto \mathcal{P}(V)$ a mapping. The pair (G, r) is called* two power level graph *and the* min-max-power graph *$G(U) := (V, E \cup E_{\max}(U))$ is defined by*

$$E_{\max}(U) := \{\{u, v\} \mid u, v \in U \quad \wedge \quad u \in r(v) \quad \wedge \quad v \in r(u)\}.$$

The graph $G(\emptyset)$ is called min-power graph *and the graph $G(V)$ is called* max-power graph. *For convenience, we assume that $E_{\max} \cap E = \emptyset$ for the remainder of this paper.*

The range assignment problem we consider can now be defined as follows.

> TWO POWER LEVEL VERTEX DEGREE (TPLVD)
>
> *Given:* A two power level graph (G, r) with $G = (V, E)$ and a positive integer $\Delta \in \mathbb{N}$.
>
> *Task:* Compute a minimum subset $D \subseteq V$ such that $\delta_{G(D)}(v) \geq \Delta$ for all vertices $v \in V$.

In the following section we will present and analyze an approximation algorithm for TWO POWER LEVEL VERTEX DEGREE. This algorithm is based on known approximation algorithms for the well-known MIN SET MULTICOVER problem.

> MIN SET MULTICOVER (MSMC)
>
> *Given:* A finite set U, a collection $\mathcal{S}$ of n subsets of U $\mathcal{S} = \{S_i \mid 1 \leq i \leq n \quad \wedge \quad S_i \subseteq U\}$ and a mapping $d : U \mapsto \mathbb{N}$ that assigns a non-negative integer to each element in U.
>
> *Task:* Compute a subset $C \subseteq \mathcal{S}$ of minimum cardinality such that each element of U is contained in at least $d(u)$ sets of C, i.e. $\forall u \in U : \quad |\{S \in C \mid u \in S\}| \quad \geq \quad d(u)$.

3. Approximation algorithm

An implementation of the approximation algorithm in pseudo-code is given in Algorithm 1. The general idea is based on the observation that every solution for TPLVD has to contain all vertices that have insufficient degree in the min-power graph, because all of these nodes definitely need additional outgoing communication links. However, to increase the (undirected) degree of these nodes it is also necessary to realize the reverse direction of outgoing communication links by increasing the transmission power of the target nodes. Choosing a minimum set of such target nodes that increases the degrees of all nodes to at least Δ can be formulated as an instance of the MIN SET MULTICOVER problem, which can then be approximated by a suitable approximation algorithm.

Observation 1. *Let $((G, r), \Delta)$ with $G = (V, E)$ be an instance for TPLVD and $T := \{v \in V \mid \delta_{G(\emptyset)}(v) < \Delta\}$ the set of vertices with insufficient vertex degree in the min-power graph $G(\emptyset)$. Then it holds, for every solution D, that $T \subseteq D$.*

Algorithm 1 Approximation Algorithm for TPLVD.

```
function APPROXTPLVD(V, E, r, Δ)
    T ← {v ∈ V | δ_G(∅)(v) < Δ}              ▷ nodes with insufficient degree
    R ← {v ∈ V | δ_G(T)(v) < Δ}                        ▷ obviously R ⊆ T
    S ← ∅                                     ▷ collection S of MSMC instance
    for v ∈ V do
        if v ∈ R then                     ▷ v has insufficient degree in G(T)
            d(v) ← Δ − δ_G(T)(v)           ▷ v needs to be covered d(v) times
        else                                ▷ v has sufficient degree in G(T)
            d(v) ← 0                          ▷ v does not need to be covered
            if v ∉ T then                          ▷ v uses min-power so far
                R_v ← {u ∈ R | {u, v} ∉ E ∧ v ∈ r(u) ∧ u ∈ r(v)}
                S ← S ∪ {R_v}           ▷ v can increase degree of these nodes
            end if
        end if
    end for
    C ← APPROXMSMC(R, S, d)                     ▷ approximate MSMC instance
    return T ∪ C
end function
```