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The number of spanning trees of a class of self-similar fractal models

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ABSTRACT

Computing the number of spanning trees of any general network model (graph) is almost too difficult to have no possibilities. As a vital constant of network model (graph), spanning trees number plays an important role not only on understanding some structural features, but also on determining some relevant dynamical properties, such as network security, random walks and percolation. Therefore, it becomes an interesting and attractive task taken more attention from various disciplines, including graph theory, theoretical computer science, physics and information science as well chemistry and so on. In this paper, our aim is to study the number of spanning trees of a class of self-similar fractal models. Firstly, we present the self-similar fractal model $N(t)$ motivated from the well-known Koch curve. Next, due to its unique growth process and its spacial topological structure, we not only compute its average degree, which shows our model is sparse, but also capture a precise analytical solution for the total number of spanning trees of model $N(t)$ by using both induction and iterative computational method. Finally, we obtain an approximate numerical value of its spanning tree entropy and then compare this value with ones of other network models researched previously. To conclude our work, we provide an opening and challenging problem that might be solved in the next coming days.

1. Introduction

As we all know, in the past several decades, there has been a growing hot wave about studies and researches of both complex systems and complex networks, which sweeps across variety of fields, from computer science, applied mathematics, applied graph theory, physics to chemistry, even to social science and biology, and the like. For instance, the World-Wide-Web (WWW) [1–3] where vertex is Web document and edge present URL link, the collaboration networks whose vertices are published articles and edges are citations [4,5], the sexual contact networks [6], protein interaction networks [7], metabolic networks [8,9] and so on. With the rapid growth of such researches about topological structure characters and dynamical properties on complex networks, amounts of artificial network models and real-life networks all have scale-free feature, which obeys the power-law $P(k) \sim k^{-\gamma}$, and small-world property, namely, smaller diameter and lower average path length. In order to probe some new topological measur-

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ing metrics and understand another structural properties on network models, as two discussed objects crossing between complex network and graph theory, the number of spanning trees and spanning trees entropy become so prevailing and popular that lots of researchers pay more attention to calculate values of both two parameters on some deterministic models, in particular on scale-free and small-world models. After that, people obtain numbers of results based on kinds of computational methods [10–16]. In addition, there are researchers taking into consideration network topological structure entropy [17] and network robustness [18,19], etc.

There is indeed a long study history about computing the solution to the total number of spanning trees on a graph, and hence this is an historical, interesting and fascinating problem in all kinds of ares, such as graph theory (a branch of mathematics), physics, biology, theoretical computer science and chemistry, and so forth. As a topological parameter of network model, spanning trees number is an important invariant correlated to several kinds of dynamical functions, including reliability [20], synchronization capability [21], random walks [22–24], to name just a few. As known, calculating the number of spanning trees of any finite graph had been theoretically answered by the famous Kirchhoff's matrix-tree theorem, which numerical value is equal to the product of all nonzero eigenvalues of the Laplacian matrix of the graph [25–27]. To guarantee the security of an enormous network model with hundreds and thousands of vertices and edges, as another tentative measuring index, calculating its spanning trees numerical values will be a demanding and troublesome task.

This paper is constructed as follows. In Sec 2, we propose a class of self-similar fractal graphs motivated from Koch curve, called $N(t)$. By computing the value of its average degree, it is proved to be a sparse graph having lower average degree. Based on the self-similarity, we give an exact analytical solution for the total number of spanning trees of our model $N(t)$ in Sec 3. And then we obtain its spanning trees entropy and compare this value with ones of another network models. In the last part of this paper, we make a brief summary and bring a meaningful and undetected problem to be taken over in the future.

Definition 1. An operation is called $V-E$ function, shorted as $f(V-E)$, if we just link two endpoints u_1 and u_2 of an edge u_1u_2 with a fixed vertex v by two new edges u_1v and u_2v, see Fig. 1(a). Similarly, another is called $E-V$ function, denoted by $f(E-V)$, if we not only link a new vertex u with two endpoints v_1 and v_2 of a stable edge v_1v_2 by two new edges uv_1 and uv_2, but also remove that old edge v_1v_2, shown in Fig. 1(b).

Definition 2. A spanning subgraph G' of graph G is a subgraph having the same vertex set as G and a number of edges E' such that $|E'| \le |E|$. A spanning tree T' of a connected G is a subgraph which is a tree with $|E'| = |V| - 1$ [25].

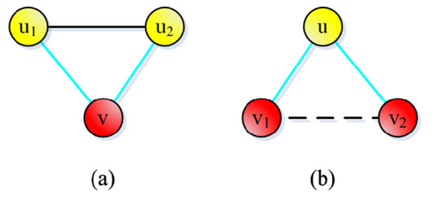

Fig. 1. The diagrams of function $f(V-E)$ and $f(E-V)$.

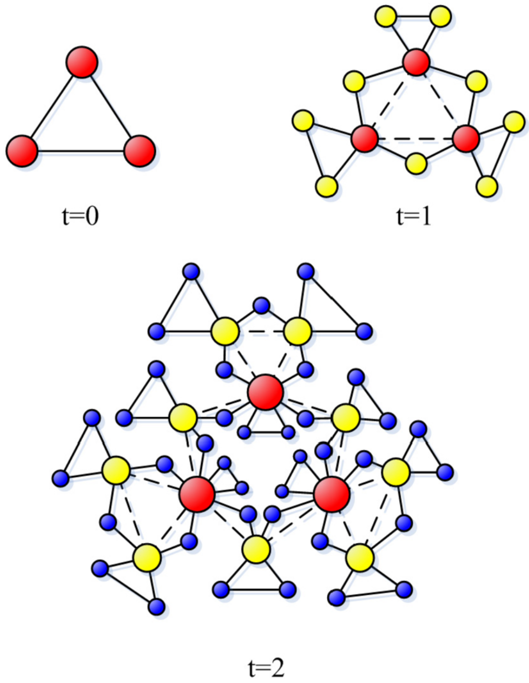

Fig. 2. The diagrams of $N(t)$ at $t = 0, 1, 2$.

2. Construction and properties of models $N(t)$

In this section, first part, the construction process of this self-similar fractal graph will be displayed by using an iterative manner, shown in Fig. 2. Second part, its some topological properties will be studied.

For $t = 0$, $N(0)$ is a C_3 which has three vertices and three edges, that is to say a so-called complete graph K_3.

For $t = 1$, $N(1)$ is obtained from $N(0)$ by taking two following operations

(1) Each vertex of $N(0)$ is applied the $f(V-E)$ only one time;
(2) Every edge of $N(0)$ is applied the $f(E-V)$ only once.

For $t \ge 2$, $N(t)$ is obtained from $N(t-1)$ by taking two operations described above.

These replication and connection procedures can be repeated indefinitely. We will use the following notations [25]: $V(t)$ and $E(t)$ denote, respectively, the set of vertices and edges of model $N(t)$. It is convenient for us to say $v_t = |V(t)|$ and $e_t = |E(t)|$. Next, we will compute v_t and e_t of our model. Initially, $v_0 = 3$, $e_0 = 3$. As $t = 1$, $v_1 = 2v_0 + v_0 + e_0 = 12$, $e_1 = 3v_0 + 2e_0 = 15$. With time