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The pessimistic diagnosability of data center networks

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ABSTRACT

A system is t/t-diagnosable if, provided the number of faulty processors is bounded by t, all faulty processors can be isolated within a set of size at most t with at most one fault-free processor mistaken as a faulty one. The pessimistic diagnosability of a system G, denoted by $t_p(G)$, is the maximal number of faulty processors so that the system G is t/t-diagnosable. Data centers are critical to the business of companies such as Amazon, Google, Facebook, and Microsoft. Based on data centers, the data center networks $D_{n,k}$, given in 2008, has many desirable features. In this paper, by exploring the structure of $D_{k,n}$, we firstly determined the pessimistic diagnosability of $D_{k,n}$ and prove that $t_p(D_{k,n}) = n+2k-2$ for $k \geq 2$ and $n \geq 2$ under the PMC model. Then we prove that $D_{k,n}$ $(k \geq 0, n \geq 2)$ is not edge transitive except the cases of $k = 0$ and $k = 1, n = 2$.

1. Introduction

In many parallel computer systems, processors are connected based on an interconnection network. An interconnection network can be modeled as an undirected simple graph with processors and links between processors as vertices and edges, respectively. In a large-scale processing systems, failures of processors are inevitable. So processor fault identification plays an important role for reliable computing. The process of identifying faulty processors is called the *diagnosis of the system*. The *diagnosability* of a system is the maximal number of faulty processors that the system can guarantee to diagnose. A system is said to be *t-diagnosable* if all faulty units can be identified provided the number of faulty units present does not exceed t.

For the purpose of self-diagnosis of a system, some different models have been proposed. Among these models, the most popular is the PMC model which is proposed by Preparata, Metze and Chen [10]. In the PMC model, every processor can test the processor that is adjacent to it and only the fault-free processor can guarantee reliable outcome. The pessimistic diagnosis strategy proposed by Kavianpour and Friedman [9] is a classic strategy based on the PMC model. In this strategy, all faulty processors to be isolated within a set having at most one fault-free processor.

Definition 1. A system is *t/t-diagnosable* if, provided the number of faulty processors is bounded by t, all faulty processors can be isolated within a set of size at most t with at most one fault-free processor mistaken as a faulty one. The *pessimistic diagnosability* of a system G, denoted by $t_p(G)$, is the maximal number of faulty processors so that the system G is t/t-diagnosable.

Many large data centers are being built to provide increasingly popular online application services, such as search, e-mails, IMs, web 2.0, and gaming, etc. Guo et al. [4] introduced the data center networks, simply say DCell, $D_{n,k}$ for computing systems, which has many desirable features for data center networking. DCell focus on the networking infrastructure inside a data center, which connects a large number of servers via high-speed links and switches. Based on data centers as a server centric in-

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terconnection network structure, DCell offers much higher network capacity compared with the tree-based, current practice. Traffic carried by DCell is distributed quite evenly across all links; there is no severe bottleneck. DCell can support millions of servers with high network capacity by only using commodity switches.

The pessimistic diagnosability of some interconnection networks have been explored. For examples, the pessimistic diagnosability of the alternating group graph AG_n and the hypercube-like network (BC network) were obtained by Tsai in [12] and [13], respectively. Wang et al. [15] studied the diagnosability of the k-ary n-cubes Q_n^k using the pessimistic strategy. Hao et al. [6] gave the pessimistic diagnosabilities of some general regular graphs which have the property that the maximum number of common neighbors for any two distinct vertices is at most two. More results about the diagnosability and the pessimistic diagnosability can be seen [2], [3], [5], [7], [8], [15], [16], [17] etc. Although there are much rich results about properties and the pessimistic diagnosability of many interconnection networks, little is known about the data center networks.

In this paper, by exploring the structure of $D_{k,n}$, we firstly prove the edge transitivity of $D_{k,n}$; Then we determined the pessimistic diagnosability of $D_{k,n}$ and prove that $t_p(D_{k,n}) = n + 2k - 2$ for $k \geq 2$ and $n \geq 2$ under the PMC model.

The remainder of this paper is organized as follows. Section 2 introduces some necessary notations and the structure properties of $D_{k,n}$. The pessimistic diagnosability of $D_{k,n}$ under PMC model is given in Section 3. The result that $D_{k,n}$ ($k \geq 0, n \geq 2$) is not edge transitive except $D_{0,n}$ for $n \geq 1$ and $D_{1,2}$ were given in Section 4. Section 5 concludes the paper.

2. Preliminaries

2.1. Notations

We use a graph, denoted by $G = (V(G), E(G))$, to represent an interconnection network, where a vertex $u \in V(G)$ represents a processor and an edge $(u, v) \in E(G)$ represents a link between vertices u and v. Throughout this paper, all graphs are finite, connected, simple and undirected. For a vertex $u \in V(G)$, let $N_G(u)$ (or simply denoted by $N(u)$) denote a set of vertices in G adjacent to u. For a vertex set $U \subseteq V(G)$, the *neighborhood* of U in G is defined as $N_G(U) = \bigcup_{v\in U} N_G(v) - U$. If $|N_G(u)| = k$ for any vertex in G, then G is *k-regular*. A *path* $P = (v_0, v_1, \ldots, v_k)$ for $k \geq 2$ is a sequence of distinct vertices such that any two consecutive vertices are adjacent, and v_0 and v_k are the *end-vertices* of the path. A path $P = (v_0, v_1, \ldots, v_k)$ forms a *cycle* if $v_0 = v_k$ and $k \geq 2$. The number of edges of a path (cycle) is its *length* and a path (cycle) of length k is called a *k-path* (*k-cycle*).

Let G be a connected graph, if $G - S$ is still connected for any $S \subseteq V(G)$ with $|S| \leq k - 1$, then G is *k-connected*. The *connectivity* of a graph G, denoted by $\kappa(G)$, defined as the minimum number of vertices whose removal results in a disconnected or trivial graph. A graph H is a *subgraph* of a graph G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. The *components* of a graph G are its maximally connected subgraphs. A component is *trivial* if it has only one vertex; otherwise, it is *nontrivial*. For any two vertices u and v in G, let $cn(G; u, v)$ denote the number of vertices who are the neighbors of both u and v, that is, $cn(G; u, v) = |N_G(u) \cap N_G(v)|$. Let $cn(G) = \max\{cn(G; u, v) : u, v \in V(G)\}$.

A graph is *vertex-transitive* if it has an automorphism mapping any given vertex to any other given vertex. A graph is *edge-transitive* if given any two edges (a, b) and (a', b'), either there is an automorphism mapping a to a' and b to b' or there is an automorphism mapping a to b' and b to a'.

2.2. The definition and properties of data center networks $D_{k,n}$

Given a positive integer m, we use $\langle m\rangle$ and $[m]$ to denote the sets $\{0, 1, 2, \ldots, m\}$ and $\{1, 2, \ldots, m\}$, respectively. For any integers $k \geq 0$ and $n \geq 2$, we use $D_{k,n}$ denote a k-dimensional DCell with n-port switches. $D_{0,n}$ is a complete graph on n vertices. We use $t_{k,n}$ to denote the number of vertices in $D_{k,n}$ with $t_{0,n} = n$ and $t_{i,n} = t_{i-1,n} \times (t_{i-1,n} + 1)$, where $i \in [k]$. Let $I_{0,n} = \langle n - 1\rangle$ and $I_{i,n} = \langle t_{i-1,n}\rangle$ for any $i \in [k]$. Then, let $V_{k,n} = \{u_k u_{k-1} \cdots u_0 |\ u_i \in I_{i,n}$ and $i \in \langle k\rangle\}$, and $V^{\ell}_{k,n} = \{u_k u_{k-1} \cdots u_\ell |\ u_i \in I_{i,n}$ and $i \in \{\ell, \ell+1, \ldots, k\}$ for any $\ell \in [k]\}$. Clearly, $|V_{k,n}| = t_{k,n}$ and $|V^{\ell}_{k,n}| = t_{k,n}/t_{\ell-1,n}$. The definition of $D_{k,n}$ is as follows [4].

Definition 2. The data center network $D_{k,n}$ is a graph with vertex set $V_{k,n}$, where a vertex $u = u_k u_{k-1} \cdots u_i \cdots u_0$ is adjacent to a vertex $v = v_k v_{k-1} \cdots v_i \cdots v_0$ if and only if there is an integer ℓ with

(1) $u_k u_{k-1} \cdots u_\ell = v_k v_{k-1} \cdots v_\ell$,
(2) $u_{\ell-1} \neq v_{\ell-1}$,
(3) $u_{\ell-1} = v_0 + \sum_{j=1}^{\ell-2}(v_j \times t_{j-1,n})$ and $v_{\ell-1} = u_0 + \sum_{j=1}^{\ell-2}(u_j \times t_{j-1,n}) + 1$ with $\ell > 1$;

or $u_k \neq v_k$, $u_k \leq v_k$ and $u_k = v_0 + \sum_{j=1}^{k-1}(v_j \times t_{j-1,n})$ and $v_k = u_0 + \sum_{j=1}^{k-1}(u_j \times t_{j-1,n}) + 1$.

$D_{0,2}$ is an edge; $D_{1,2}$ is a cycle of length 6. $D_{2,2}$ is shown in Fig. 1. It is clear that $D_{k,n}$ is a regular graph with $t_{k,n}$ vertices.

For any integer $d \geq 0$, when two adjacent vertices u and v have a leftmost differing element at the position d, denoted by $\text{ldiff}(u, v) = d$. For any $\alpha \in V^{\ell}_{k,n}$ with $\ell \in [k]$, we use $D^{\alpha}_{\ell-1,n}$ to denote the graph obtained by prefixing the label of each vertex of one copy of $D_{\ell-1,n}$ with α. Clearly, $D_{\ell-1,n} \cong D^{\alpha}_{\ell-1,n}$. For any integers $n \geq 2$ and $k \geq 1$, edges joining vertices in the same copy of $D_{k-1,n}$ are called *internal edges* and edges joining vertices in disjoint copies of $D_{k-1,n}$ are called *external edges*. Clearly, each vertex of $D^{i}_{k-1,n}$ is joined to exactly one external edge and $(n + k - 2)$-internal edges for each $i \in I_{k,n}$.

From the definition of $D_{k,n}$ in [4], the following Properties 1 can be gotten directly.