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A novel Trefftz method for solving the multi-dimensional direct and Cauchy problems of Laplace equation in an arbitrary domain

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Abstract

We propose a novel Trefftz method for the numerical solution of the direct problem as well as the Cauchy problem of the multi-dimensional Laplace equation in an arbitrary domain. In the multiple/scale/direction Trefftz method (MSDTM) the directions are hyper-spherical unit vectors given explicitly, and the scales are determined by the collocation points on the boundary which play a role of post-conditioner regularization of the resultant linear system to overcome the highly ill-posed behavior of the inverse Cauchy problem. The present paper can largely reduce the number of unknown coefficients appeared in the series expansion of numerical solution. Several three-dimensional numerical examples of direct problems, including a non-harmonic boundary value problem, and the inverse Cauchy problems in irregular domains are solved to demonstrate the efficiency and accuracy of the MSDTM, where the noises up to the levels 10%-30% are imposed on the given data to test the robustness.

Keywords: Multi-dimensional Laplace equation, Multiple/scale/direction Trefftz method, Irregular domain, Inverse Cauchy problem

1. Introduction

In this paper we extend the Trefftz method for the two-dimensional Laplace equation to a novel yet simple Trefftz method for the q-dimensional Laplace equation:

$$\Delta u(\mathbf{x}) = 0, \quad \mathbf{x} \in \Omega \subset \mathbb{R}^q, \quad q \geq 3, \tag{1}$$

$$u|_{\mathbf{x}\in\Gamma_1} = g(\mathbf{x}), \tag{2}$$

$$\nabla u(\mathbf{x}) \cdot \mathbf{n}|_{\mathbf{x}\in\Gamma_2} = h(\mathbf{x}), \tag{3}$$

where Δ is the q-dimensional Laplacian operator, and Ω is an arbitrary q-dimensional bounded domain enclosed by a smooth boundary Γ, whose normal direction is denoted by $\mathbf{n}$. When $\Gamma_1 \cup \Gamma_2 = \Gamma$ and $\Gamma_1 \cap \Gamma_2 = \varnothing$ we encounter a direct problem of the q-dimensional Laplace equation; otherwise, we encounter the generalized inverse boundary value prob-