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Existence and global exponential stability of anti-periodic solutions of high-order bidirectional associative memory (BAM) networks with time-varying delays on time scales☆

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ABSTRACT

In this paper, high-order bidirectional associative memory (BAM) networks with time-varying delays on time scales are investigated. With the help of the time scale calculus theory, differential inequality theory, some analysis skills and the Liapunov functional method, a set of sufficient conditions ensuring the existence and exponential stability of anti-periodic solutions for the high-order BAM neural network with time-varying delays are obtained. Our results are new and complement some previously known ones. Moreover, a numerical example is presented to verify the theoretical results.

1. Introduction

Since high-order neural networks possess stronger approximation property, fast convergence rate, greater storage capacity, and higher fault tolerance than lower-order neural networks, they have been successfully applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision and image processing. Considering that the applicability and efficiency of such networks hinges upon their dynamics, the investigation of dynamical behaviors is a necessary step for practical design and application of the neural networks. Recently, considerable effort has been devoted to the dynamical behaviors such as the existence and stability of the equilibrium point, periodic and almost periodic solutions of high-order bidirectional associative memory (BAM) networks with time-varying delays and continuously distributed delays (see [1–6]). It is expected to obtain a deep and clear understanding of the dynamics of complicated neural networks with delays. We know that the signal transmission process of neural networks can often be described as an anti-periodic solution process. Thus the existence and stability of anti-periodic solutions are an important topic in characterizing the behavior of nonlinear differential equations [7–20]. Therefore it is worth while to investigate the existence and stability of anti-periodic

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solutions for BAM neural networks. In 2013, Li et al. [21] investigated the following high-order BAM neural networks with delays on time scales

$$\begin{cases} x_i^{\triangle}(t) &= -a_i(t)x_i(t) + \sum_{j=1}^{m} b_{ij}(t)g_j(y_j(t-\tau_j(t))) + \sum_{j=1}^{m}\sum_{l=1}^{m} e_{ijl}(t)p_j(y_j(t-\tau_j(t)))q_l(y_l(t-\tau_l(t))) + I_i(t), \\ y_j^{\triangle}(t) &= -d_j(t)y_j(t) + \sum_{i=1}^{n} c_{ji}(t)f_i(x_i(t-\omega_i(t))) + \sum_{i=1}^{n}\sum_{l=1}^{n} s_{jil}(t)v_i(x_i(t-\omega_i(t)))\omega_l(x_l(t-\omega_l(t))) + J_j(t), \end{cases} \tag{1.1}$$

where $t \in \mathbb{T}(t > 0)$, $\mathbb{T}$ is a periodic time scale, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $x_i(t)$ and $y_j(t)$ denote the potential (or voltage) of the cell i and j at time t, $a_i(t)$ and $d_j(t)$ denote the rate with which the cell i and j reset their potential to the resting state when isolate from the other cells and inputs, time $\tau(t)$ and $\omega(t)$ are non-negative, they correspond to finite speed of axonal signal transmission, b_{ij}, c_{ji}, e_{ijl} and s_{jil} are the first- and second-order connection weights of the neural network, respectively, I_i and J_j stand for the ith and the jth component of an external input source that introduce from outside the network to the cell i and j, respectively. Applying the fixed point theorem and constructing a suitable Lyapunov functional, Li et al. [21] obtained some sufficient conditions to ensure the existence and global exponential stability of almost periodic solution of the model (1.1) on time scales. The main aim of this article is to establish some sufficient conditions for the existence and exponential stability of anti-periodic solutions of (1.1). To the best of our knowledge, it is the first time to focus on the stability and existence of anti-periodic solutions of (1.1) on time scales.

For the sake of simplicity, set $[a, b]_{\mathbb{T}} := \{t \in \mathbb{T} : a \le t \le b\}$ and assume that $0 \in \mathbb{T}$, $\mathbb{T}$ is unbounded above, i.e., $\sup \mathbb{T} = \infty$. What·s more, we will use $x = (x_1, x_2, \ldots, x_k)^T \in \mathbb{R}^k$ to denote a column vector, in which the symbol $(.)^T$ denotes the transpose of a vector. Let $|x|$ be the absolute-value vector given by $|x| = (|x_1|, |x_2|, \ldots, |x_k|)$, and define $\|x\| = \sum_{i=1}^{k}|x_i|$.

Let $u(t) = (x_1(t), x_2(t), \ldots, x_n(t), y_1(t), y_2(t), \ldots, y_m(t))^T \in C(\mathbb{T}, \mathbb{R}^{n+m})$, $u(t)$ is said to be ω-anti-periodic on $\mathbb{T}$, if $x_i(t+\omega) = -x_i(t)$, $y_j(t+\omega) = -y_j(t)$ for all $t \in \mathbb{T}$, $t + \omega \in \mathbb{T}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. The initial conditions of (1.1) are of the form

$$\begin{cases} x_i(s) = \varphi_i(s), s \in [-\tau, 0]_{\mathbb{T}}, \tau = \max_{1\le j\le m}\{\tau_j\}, i = 1, 2, \ldots, n, \\ y_j(s) = \psi_j(s), s \in [-\delta, 0]_{\mathbb{T}}, \delta = \max_{1\le i\le n}\{\omega_i\}, j = 1, 2, \ldots, m, \end{cases} \tag{1.2}$$

where $\varphi_i \in C([-\tau, 0]_{\mathbb{T}}, \mathbb{R})$, $\psi_j \in C([-\delta, 0]_{\mathbb{T}}, \mathbb{R})$.

For convenience, we introduce some notations as follows

$$\bar{I}_i = \sup_{t\in\mathbb{T}}|I_i(t)|, \bar{I} = \max_{1\le i\le n}\{\bar{I}_i\}, \bar{J}_j = \sup_{t\in\mathbb{T}}|J_j(t)|, \bar{J} = \max_{1\le j\le m}\{\bar{J}_j\}, \underline{a}_i = \inf_{t\in\mathbb{T}}|a_i(t)|,$$
$$\bar{b}_{ij} = \sup_{t\in\mathbb{T}}|b_{ij}(t)|, \bar{e}_{ijl} = \sup_{t\in\mathbb{T}}|e_{ijl}(t)|, \bar{c}_{ji} = \sup_{t\in\mathbb{T}}|c_{ji}(t)|, \bar{s}_{jil} = \sup_{t\in\mathbb{T}}|s_{jil}(t)|, \underline{d}_j = \inf_{t\in\mathbb{T}}|d_j(t)|,$$

where $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $l = 1, 2, \ldots, n$.

Denote $\mathbb{R}^+ = (0, \infty)$, $\mathbb{T}^+ = (0, \infty)_{\mathbb{T}}$. Throughout this paper, for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, $l = 1, 2, \ldots, n$, it will be assumed that

H 1.

$$a_i, d_j \in C(\mathbb{T}, \mathbb{R}^+), b_{ij}(t+\omega)g_j(u) = -b_{ij}(t)g_j(-u), e_{ijl}(t+\omega)p_j(u)q_l(v) = -e_{ijl}(t)p_j(-u)q_l(-v),$$

$$c_{ji}(t+\omega)f_i(u) = -c_{ji}(t)f_i(-u), s_{jil}(t+\omega)v_i(u)\omega_l(v) = -s_{jil}(t)v_i(-u)\omega_l(-v).$$

H 2. There exist nonnegative constants L_{jg}, L_{jp}, L_{lq}, L_{if}, L_{iv} and $L_{l\omega}$ such that

$$|g_j(u) - g_j(v)| \le L_{jg}|u-v|, |p_j(u) - p_j(v)| \le L_{jp}|u-v|, |q_l(u) - q_l(v)| \le L_{lq}|u-v|,$$
$$|f_i(u) - f_i(v)| \le L_{if}|u-v|, |v_i(u) - v_i(v)| \le L_{iv}|u-v|, |\omega_l(u) - \omega_l(v)| \le L_{l\omega}|u-v|.$$

H 3. There exist constants $\eta > 0$ and $\gamma > 0$ such that

$$-\underline{a}_i\gamma + \sum_{j=1}^{m}\bar{b}_{ij}L_{ig}\gamma + \sum_{j=1}^{m}\sum_{l=1}^{m}\bar{e}_{ijl}L_{jp}L_{lq}\gamma^2 + \bar{I}_i < -\eta < 0,$$
$$-\underline{d}_j\gamma + \sum_{i=1}^{n}\bar{c}_{ji}L_{if}\gamma + \sum_{i=1}^{n}\sum_{l=1}^{n}\bar{s}_{jil}L_{iv}L_{i\omega}\gamma^2 + \bar{J}_j < -\eta < 0.$$

The remainder of the paper is organized as follows. In Section 2, we introduce some notations and definitions, and state some preliminary results which are needed in later sections. In Section 3, we establish our main results for the existence and exponential stability of anti-periodic solutions of (1.1). In Section 4, we present an example to illustrate the feasibility and effectiveness of our results obtained in previous sections.