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Topological analysis of voxelized objects by discrete geodesic Reeb graph ☆

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ABSTRACT

We introduce here the concept of *discrete level sets* (DLS) that can be constructed on a voxelized surface with the assurance of certain topological properties. This eventually aids in construction of *discrete geodesic Reeb graph* (DGRG) on a voxelized object, for topological analysis. Under various transformations like rotation and topology-constrained anisotropic deformation, a DGRG remains invariant to typical topological features like loops or cycles, which eventually helps in identifying 'handles' in the underlying object. Experiments on different datasets show promising results on the practical usefulness of DLS and DGRG towards extraction of high-level topological features of arbitrary voxel sets.

1. Introduction

Voxelization today is not only important in the field of object discretization and representation but also gaining remarkable progress in additive manufacturing through rapid prototyping (RP) techniques like stereo-lithography, 3D printing, and fused deposition modeling [18,29–31,44]. Hence, the collection of work related to voxelization, as seen in today's literature, can be divided into two categories—one covering the theories and algorithmic solutions for object discretization and another dealing with different RP techniques using digital technology. The latter category mostly relies on a *digital building matter* in the sense that the building block is a digital unit or *voxel*, as opposed to the analog (continuous) material used in conventional RP [13,27,28,42,48].

Whether the subject relates to analytical discretization or relates to physical manufacturing, the underlying theory or methodology of voxelization has a strong impact on the consistency or on the solidity of the resultant product. In either case, these characteristics can be analyzed well in the purview of discrete geometry and topology, as a collection of voxels is usually obtained by a particular process in a certain theoretical framework [35,41].

In our work, we focus on this with a two-fold objective—first to show how a surface should be voxelized for its readiness to construction of *discrete level sets* (DLS) and then to establish their usefulness in extraction of high-level topological information through a *discrete geodesic Reeb graph* (DGRG).

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1.1. Existing work

We give here a brief review of the development and the state-of-the art practices related to voxelization, computation of discrete geodesics and iso-contours or level sets, and construction and analysis of Reeb graph.

1.1.1. Voxelization

The early work on physical modeling of a surface or volume element can be seen in [29,44,45] and in the articles referred to therein. Those work, however, did not address the topological issues related to voxelization. The theoretical frameworks along with the topological issues came up gradually in a later stage. For example, in [16], some of the topological properties were discussed, which included holes, cavities, simple points, separability, and penetration.

With the growing need for digitization and cutting-edge technology, different techniques for voxelization have been proposed off and on, taking into account different apparatus, computational models, cost factors, and product requirement. A low-cost methodology based on z-buffer and multi-view depth information is developed in [32]. To incorporate an anti-aliasing effect during voxel rendition, a multi-resolution technique is proposed in [17]. For adding more features available in graphics workstations, such as texture mapping and frame-buffer blending functions, a hardware-accelerated approach is shown to be effective in [25]. The idea of exploiting programmable graphics hardware is also used in [21] for voxelization of a polygonal model after mapping it into three sheet buffers and then synthesizing into a single worksheet recording the volumetric representation of the target.

Voxelization is also useful for simplification and repair of a polygonal model, as shown in [43], with 3D morphological operations on the scan-converted voxel set. Further, with the emergence of GPU functionalities, a variety of applications with voxelized objects have come up in recent time. For example, in [36], a GPU-accelerated approach is proposed for creation of multi-valued solid volumetric models with different solid slice functions and material description in order to make it useful for different applications like collision detection, medical simulation, volume deformation, 3D printing, and computer art. In [24], a filtering algorithm is designed to build a density estimate for deduction of normals from the voxelized model, which is shown to be useful in simulation of translucency effects and particle interactions. In fact, very recently, many such real-time simulations and applications are shown to be efficiently realizable when a voxelized dataset is used; these include urban modeling [51], octree-based sparse voxelization for 3D animation [20], fluid simulation with dynamic obstacles [56], discrete radiosity [37], light refraction and transmittance in complex scenes [14], etc.

1.1.2. Geodesics

The literature on geodesics and iso-contours, as on today, is predominantly focused on closed orientable 2-manifold surfaces, i.e., objects with triangulated-mesh representation in the Euclidean space. Hence, the techniques are mostly from differential and computational geometry; see, for example, [2,15,40,46,49,53–55]. As geodesics find various applications in remeshing, non-rigid registration, surface parametrization, shape editing and segmentation, the notion of approximate geodesic distance is also proposed recently in [54] as a practical alternative for the exact solution [53].

In the domain of voxel complexes, however, no significant work can be found on discrete geodesics, barring a few [15,20]. In [15], the concept of visibility—a well-known concept in computational geometry—is defined in the discrete space based on digital straightness. In [20], as sparse (i.e., highly disconnected) voxel set is used, the geodesic metric is based on Euclidean norm.

1.1.3. Reeb graph

Reeb graph is a topological structure given by the *quotient topology* of a topological space induced by a real-valued continuous function defined on the space. Being rooted in Morse theory, it is a powerful mathematical tool for topological analysis and conceptual visualization of geometric objects. For a 2-manifold surface (e.g., triangle mesh), the Reeb graph is usually constructed from a level set function defined on the surface [22]. Topological properties and intrinsic geometric shape of the underlying surface is studied through the set of critical points of the level set function. The rationale goes to the well-established power and ability of Morse function or level set while studying and analyzing objects in their respective continuous analytical form or discretized form [39].

A significant amount of research work has been done on construction and application of Reeb graphs [1,3,4,6,7,12,19,23,26,33,34,47,50,52], and a review can be seen in [7]. Construction of Reeb graph depends on the representation of the object (2-manifold surface, polyhedron model, etc.) and the real-valued (level set) function defined on it. For 2-manifold surfaces, most of the existing work use a height function as the level; then the critical points of bifurcation and loop are identified for detection of topological features, such as 'handles' in the underlying object [22,26,33,34,38,47,52]. In [50], continuous geodesic distance on triangle mesh is used for defining the level set function and constructing its Reeb graph.

1.2. Our contribution

As briefed in Section 1.1, a multitude of work have been carried out on geodesics and Reeb graph on 2-manifold surfaces in the 3D Euclidean space. However, geodesics on voxelized (i.e., 3-manifold) surfaces and resultant Reeb graph in the voxel space have not been studied so far. This motivates us to look into this maiden problem. We introduce the concept of *discrete level sets* (DLS) that can be constructed on a well-formed voxelized surface. We show that for an appropriate scale factor,

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