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Object digitization up to a translation

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ABSTRACT

This paper presents a study on the set of the digitizations generated by all the translations of a planar body on a square grid. First the translation vector set is reduced to a bounded subset, then the dual introduced in [1] linking the translation vector to the corresponding digitization is proved to be piecewise constant. Finally, a new algorithm is proposed to compute the digitization set using the dual.

1. Introduction

For a given planar body digitization method, the result depends on the object relative position with respect to the digitization grid. As a result, distinct digital sets can be obtained. This variability may influence the reconstruction of the geometrical and topological attributes of the planar body. For instance, conditions have been given to preserve the topology during the digitization step [9,8].

This paper focuses on the effect of the grid translations on the digitized object. This issue has been studied on some geometrical primitives, *i.e.* the straight segments and the discs. Straight segment digitizations have been discussed in function of the straight segment slope and its vertical position. The function giving the digital straight segment from these two inputs is known as the *preimage*. Several properties have been proved on the preimage and it is widely used, *e.g.* for digital straight segment recognition [2]. The number of oval and disc digitizations in function of their radius up to a translation was studied in [6,7,3]. The number of digital discs including exactly N points was treated in [4] and an asymptotic bound on this number was given in [5]. Our study follows a previous work [1] which focused on function graphs digitizations.

After introducing the dual definition in Section 2, its structure is investigated in Section 3 and it is proved to be piecewise constant. Two algorithms taking the dual as input are presented in Section 4. The first one computes the digitization of a planar body for a particular position of the grid while the second one propagates the output of the former to obtain the whole family of the digitizations.

2. Definition of the digitization dual

Let us consider a connected set S in $\mathbb{R}^2$ whose boundary is a simple closed (Jordan) curve Γ. Thanks to the Jordan curve theorem, we may assume a continuous map $f\colon \mathbb{R}^2 \to \mathbb{R}$ such that Γ, respectively S, is implicitly defined by $\Gamma = \{x \in \mathbb{R}^2 \mid f(x) = 0\}$, respectively $S = \{x \in \mathbb{R}^2 \mid f(x) \leq 0\}$.

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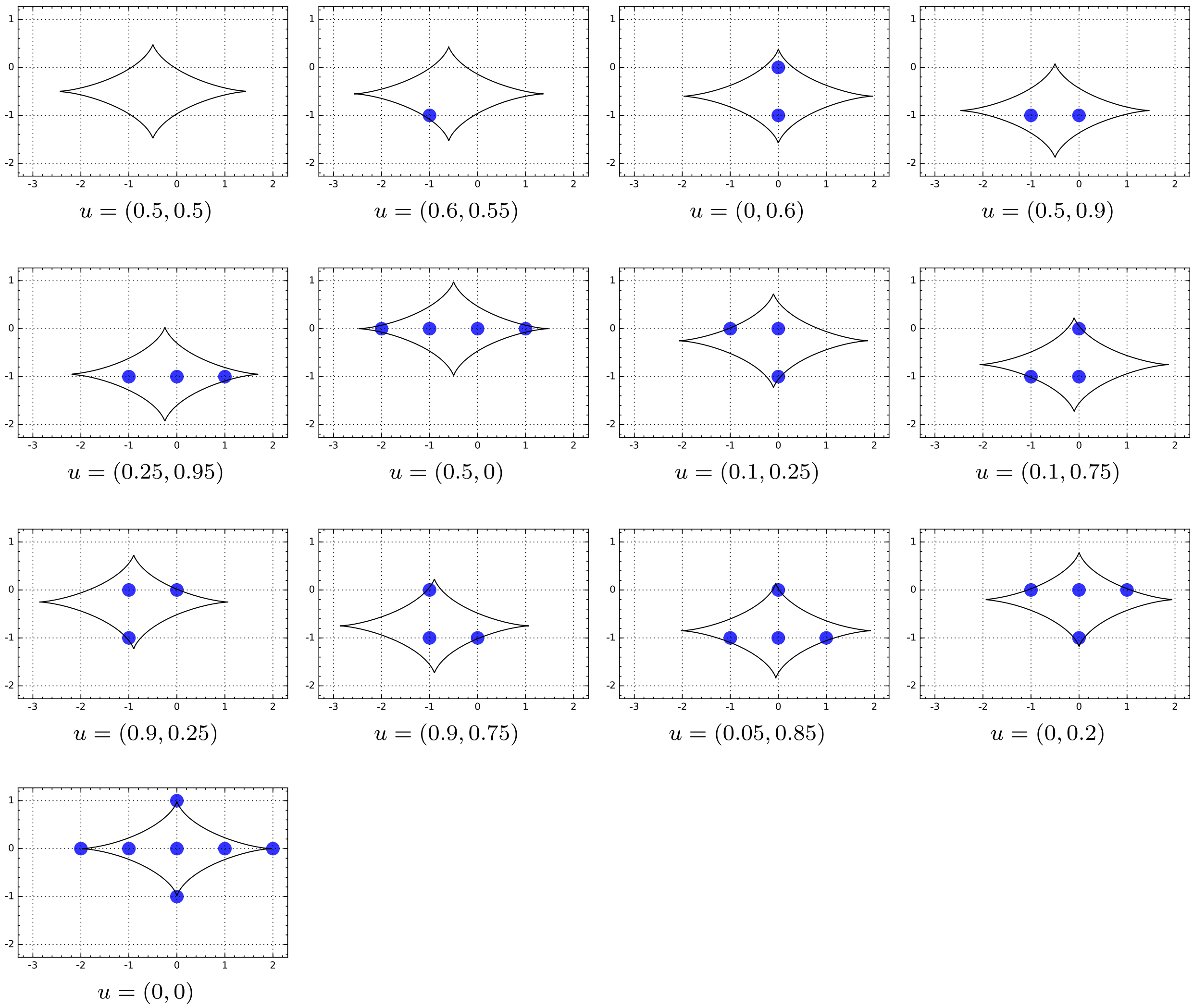

Fig. 1. The thirteen Gauss digitizations of the sets defined by $((x+u_x)/2)^{2/3}+(y+u_y)^{2/3} \leq 1$, $(u_x, u_y) \in \mathbb{R}^2$ (the first one is the empty set). The goal of the article is to give a representation of the mapping between the translation vector (u_x, u_y) and the digitization.

The common methods to model the digitization of the set S are closely related to each other. In this paper, we assume a *Gauss digitization*. This method associates to the set S the grid points that lie inside S or, equivalently, the binary image defined on $\mathbb{Z}^2$ whose 1's are the points inside S. The aim of this paper is to represent and describe the set of the Gauss digitizations of Γ obtained using the grids generated by the action of the translation group on the standard grid. Equivalently, we can consider a unique grid, the standard one, and let the group of translations acts on S. These two points of view are used in the article. Fig. 1 exhibits these Gauss digitizations for the set $\mathcal{S}_{\text{astro}}$ bounded by the “stretched” astroid

$$\left(\frac{x}{2}\right)^{2/3} + y^{2/3} = 1 \ .$$

We will use this shape to illustrate the notions and properties discussed throughout the article.

The Gauss digitization is a finite subset of $\mathbb{Z}^2$ and we are only interested in the relative positions of its elements. In other words, $\mathbb{Z}^2$ is viewed as a geometrical subset of the Euclidean plane without any preferential origin. Thus, rather than defining a mapping from the translation vectors to the digitizations of the translated body, we define a mapping $\tilde{\varphi}_S$ from the torus $\mathbb{T} = \mathbb{R}^2/\mathbb{Z}^2$ to the equivalence classes under integral translations of the digitizations (the tilde sign is used throughout the article when the function domain is the torus). This mapping $\tilde{\varphi}_S$ is called the *dual*[1] *of the digitizations*.

[1] We call it “dual” for the translations act on the digitizations while the dual of the digitizations maps the translations to the digitizations.

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