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Dual power assignment via second Hamiltonian cycle

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ABSTRACT

A *power assignment* is an assignment of transmission power to each of the wireless nodes of a wireless network, so that the induced graph satisfies some desired properties. The *cost* of a power assignment is the sum of the assigned powers. In this paper, we consider the dual power assignment problem, in which each wireless node is assigned a high- or low-power level, so that the induced graph is strongly connected and the cost of the assignment is minimized. We improve the best known approximation ratio from $\frac{\pi^2}{6} - \frac{1}{36} + \epsilon \approx 1.617$ to $\frac{11}{7} \approx 1.571$. Moreover, we show that the algorithm of Khuller et al. [11] for the strongly connected spanning subgraph problem, which achieves an approximation ratio of 1.617, is 1.522-approximation algorithm for symmetric directed graphs. The innovation of this paper is in achieving these results by using interesting conditions for the existence of a second Hamiltonian cycle.

1. Introduction

Given a set P of wireless nodes distributed in a two-dimensional plane, a *power assignment* (or a range assignment), in the context of wireless networks, is an assignment of transmission range r_u to each wireless node $u \in P$, so that the induced communication graph has some desired properties, such as strong connectivity. The *cost* of a power assignment is the sum of the assigned powers, i.e., $\sum_{u \in P} r_u^\alpha$, where α is a constant called the *distance-power gradient* whose typical value is between 2 and 5. A power assignment induces a (directed) *communication graph* $G = (P, E)$, where a directed edge (u, v) belongs to the edge set E if and only if $|uv| \le r_u$, where $|uv|$ is the Euclidean distance between u and v. The communication graph G is *strongly connected* if, for any two nodes $u, v \in P$, there exists a directed path from u to v in G. In the standard power assignment problem, one has to find a power assignment of P such that (i) its cost is minimized, and (ii) the induced communication graph is strongly connected.

When the available transmission power levels for each wireless node are continuous in a range of reals, many researchers have proposed algorithms for the strong connectivity power assignment problem [5,8,7,13,14]. In particular, 2-approximation algorithms based on minimum spanning trees were proposed in [5,13]. When the wireless nodes are deployed in the 2-dimensional or the 3-dimensional space, the problem is known to be NP-hard [7,13]. A survey covering many variations of the problem is given in [6].

In practice, it is usually impossible to assign arbitrary power levels (ranges) to the transmitters of a radio network. Instead one can only choose from a constant number of preset power levels corresponding to a constant number of ranges.

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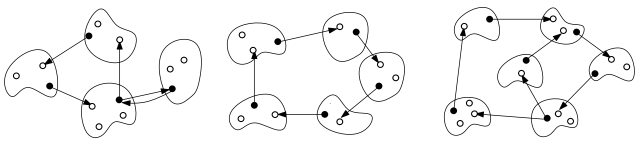

(a) (b) (c)

Fig. 1. Examples of k-contractible structures: (a) 4-contractible structure, (b) 5-contractible structure, and (c) 6-contractible structure. The solid circles in each k-contractible structure represent the nodes of the k-contracted set of the components.

In this paper we consider the dual power assignment version, also known as power-assignment with two power levels problem, in which each wireless node can transmit in one of two (*high* or *low*) transmission power levels. Let r_H and r_L denote the transmission ranges of the high- and low-transmission powers, respectively.

The cost of an assignment of power levels to the transmitters is a function of the form $r_H^c n_H + r_L^c(n - n_H)$, where n_H is the number of transmitters that are assigned range r_H and c is a constant typically between 2 to 5. Thus, the cost of an assignment is determined solely by the number of transmitters that are assigned range r_H. Moreover, a k-approximation algorithm that minimizes the number of transmitters that are assigned high range (r_H) yields a k^*-approximation algorithm for the dual power assignment problem where $k^* < k$. More precisely, our algorithm that computes $k = \frac{11}{7}$, yields that $k^* = 11r_H^c/(7r_H^c + 4r_L^c)$. For example, if $r_L = 1$, $r_H = 2$, and $c = 2$, then $k^* = \frac{11}{8}$. Therefore, the objective in the dual power assignment problem is equivalent to minimizing the number of wireless nodes that are assigned high-transmission range r_H.

The dual power assignment (DPA) problem was shown to be NP-hard [3,16]. Rong et al. [16] gave a 2-approximation algorithm, while Carmi and Katz in [3] gave a 9/5-approximation algorithm and a faster 11/6-approximation algorithm. Later, Chen et al. [4] proposed an $O(n^2)$ time algorithm with approximation ratio of 7/4. Recently, Calinescu [2] improved this approximation ratio to ≈ 1.617, using in a novel way the algorithm of Khuller et al. [12,11] for computing a minimum strongly connected spanning subgraph.

A related version asks for a power assignment that induces a connected (also called "symmetric" or "bidirected") graph. This version is also known to be NP-hard. The best known approximation algorithm is based on techniques that were applied to Steiner trees, and achieves approximation ratio of 3/2 [15].

1.1. Our results

In Section 2, we improve the best known approximation ratio for the dual power assignment problem (DPA) from $\frac{\pi^2}{6} - \frac{1}{36} + \epsilon \approx 1.617$ to $\frac{11}{7} \approx 1.571$. In Section 3, we present a conjecture regarding an interesting characterization for the existence of a second Hamiltonian cycle and its applications, and we prove the conjecture for some special cases that are utilized (i) to achieve the approximation ratio for the DPA problem mentioned above, and (ii) to show (in Section 4) that the algorithm of Khuller et al. [11], which achieves a approximation ratio of 1.617, is a 1.522-approximation algorithm for the strongly connected spanning subgraph problem in symmetric unweighted directed graphs. Moreover, in Section 4, the correctness of the conjecture would imply that the approximation algorithm of Khuller et al. is actually a 3/2-approximation algorithm in symmetric directed graphs. Finally, in Section 4.1, we show that the algorithm of Khuller et al. is a $\frac{3k-2}{2k}$-approximation algorithm for symmetric directed graphs with bounded cycle length, where $k < 24$ is the maximum cycle length in the graph.

2. Dual power assignment

Let P be a set of wireless nodes in the plane and let $G_R = (P, E_R)$ be the communication graph that is induced by assigning a high transmission range r_H to the nodes in a given subset $R \subseteq P$ and assigning low transmission range r_L to the nodes in $P \setminus R$, and with edge set $E_R = \{(u, v) : |uv| \le r_u\}$.

Definition 2.1. A ***strongly connected component*** C of G_R is a maximal subset of P, such that for each pair of wireless nodes u, v in C, there exists a path from u to v in G_R.

Definition 2.2. The ***components graph*** CG_R of G_R is an undirected graph in which there is a node C_i for each strongly connected component C_i of G_R (throughout this paper, for convenience of presentation, we will refer to the nodes of CG_R as components, and to the wireless nodes of G_R as nodes). In addition, there exists an edge between two components C_i and C_j if and only if there exist two nodes $u \in C_i$ and $v \in C_j$ such that $|uv| \le r_H$.

Definition 2.3. A set $Q \subseteq P$ is a ***k-contracted set*** of a set $\mathcal{C} = \{C_1, C_2, \ldots, C_k\}$ of k distinct components in CG_R if $|Q \cap C_i| = 1$ for each $C_i \in \mathcal{C}$, and the components in $\mathcal{C}$ are contained in the same strongly connected component in $G_{R\cup Q}$; see Fig. 1 for illustration.

Let $\mathcal{C} = \{C_1, C_2, \ldots, C_k\}$ be a set of components in CG_R, let Q be a k-contracted set of $\mathcal{C}$, and let v_i be the node in $Q \cap C_i$, for each $C_i \in \mathcal{C}$.