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Parameterized algorithms for recognizing monopolar and 2-subcolorable graphs ☆,☆☆

Iyad Kanj [a], Christian Komusiewicz [b,*], Manuel Sorge [c], Erik Jan van Leeuwen [d]

[a] School of Computing, DePaul University, Chicago, USA
[b] Institut für Informatik, Friedrich-Schiller-Universität Jena, Germany
[c] Institut für Softwaretechnik und Theoretische Informatik, TU Berlin, Germany
[d] Department of Information and Computing Sciences, Utrecht University, The Netherlands

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ABSTRACT

A graph G is a (Π_A, Π_B)-graph if $V(G)$ can be bipartitioned into A and B such that $G[A]$ satisfies property Π_A and $G[B]$ satisfies property Π_B. The (Π_A, Π_B)-Recognition problem is to recognize whether a given graph is a (Π_A, Π_B)-graph. There are many (Π_A, Π_B)-Recognition problems, including the recognition problems for bipartite, split, and unipolar graphs. We present efficient algorithms for many cases of (Π_A, Π_B)-Recognition based on a technique which we dub inductive recognition. In particular, we give fixed-parameter algorithms for two NP-hard (Π_A, Π_B)-Recognition problems, Monopolar Recognition and 2-Subcoloring, parameterized by the number of maximal cliques in $G[A]$. We complement our algorithmic results with several hardness results for (Π_A, Π_B)-Recognition.

1. Introduction

A (Π_A, Π_B)*-graph*, for graph properties Π_A, Π_B, is a graph $G = (V, E)$ for which V admits a partition into two sets A, B such that $G[A]$ satisfies Π_A and $G[B]$ satisfies Π_B. There is an abundance of (Π_A, Π_B)-graph classes, and important ones include *bipartite graphs* (which admit a partition into two independent sets), *split graphs* (which admit a bipartition into a clique and an independent set), and *unipolar graphs* (which admit a bipartition into a clique and a cluster graph). Here a *cluster graph* is a disjoint union of cliques. An example for each of these classes is shown in Fig. 1.

The problem of recognizing whether a given graph belongs to a particular class of (Π_A, Π_B)-graphs is called (Π_A, Π_B)-Recognition, and is known as a *vertex-partition* problem. Recognition problems for (Π_A, Π_B)-graphs are often NP-hard [1,13,20], but bipartite, split, and unipolar graphs can all be recognized in polynomial time [24,16,23,12,29]. With the aim of generalizing these polynomial-time algorithms, we study the complexity of recognizing certain classes of (Π_A, Π_B)-graphs, focusing on two particular classes that generalize split and unipolar graphs, respectively. To achieve our

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* Corresponding author.
E-mail addresses: ikanj@cs.depaul.edu (I. Kanj), christian.komusiewicz@uni-jena.de (C. Komusiewicz), manuel.sorge@tu-berlin.de (M. Sorge), e.j.vanleeuwen@uu.nl (E.J. van Leeuwen).

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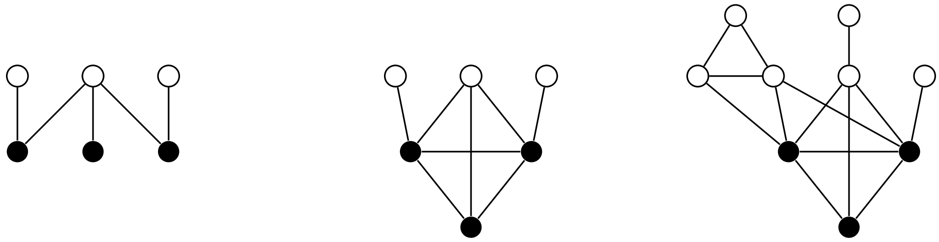

Fig. 1. Three examples of (Π_A, Π_B)-graphs, where the coloring gives a (Π_A, Π_B)-partition. The vertices of A are black and the vertices of B are white. Left: in bipartite graphs, A and B are independent sets. Center: in split graphs, A is a clique and B is an independent set. Right: in unipolar graphs, A is a clique and B induces a cluster graph.

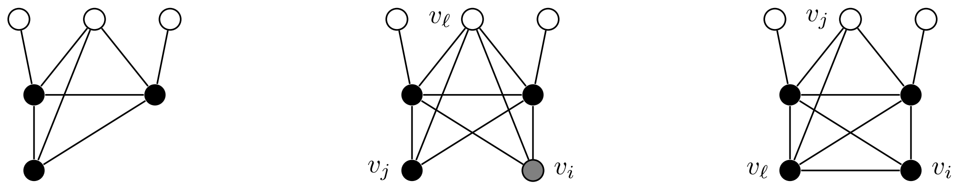

Fig. 2. An example of the inductive step in inductive recognition. Left: a split graph G_{i-1} with a given partition into a clique A and an independent set B. Center: the partition for G_{i-1} cannot be directly extended to a partition for G_i since the vertex v_i has a nonneighbor v_j in A and a neighbor v_ℓ in B. Right: after deciding to put v_i in the clique A, we can repair the partition by moving v_j to the independent set B and v_ℓ to the clique A.

goals, we formalize a technique, which we dub inductive recognition, that can be viewed as an adaptation of iterative compression to recognition problems. We believe that the formalization of this technique will be useful in general for designing algorithms for recognition problems.

Inductive recognition. The *inductive recognition* technique, described formally in Section 3, can be applied to solve the (Π_A, Π_B)-Recognition problem for certain hereditary (Π_A, Π_B)-graph classes. Intuitively, the technique works as follows. Suppose that we are given a graph $G = (V, E)$ and we have to decide its membership of the (Π_A, Π_B)-graph class. We proceed in iterations and fix an arbitrary ordering of the vertices; in the following, let $n := |V|$ and $m := |E|$. We start with the empty graph G_0, which trivially belongs to the hereditary (Π_A, Π_B)-graph class. In iteration i, for $i = 1, \ldots, n$, we recognize whether the subgraph G_i of G induced by the first i vertices of V still belongs to the graph class, assuming that G_{i-1} belongs to the graph class.

Inductive recognition is essentially a variant of the iterative compression technique [26], tailored to recognition problems. The crucial difference, however, is that in iterative compression we can always add the ith vertex v_i to the solution from the previous iteration to obtain a new solution (which we compress if it is too large). However, in the recognition problems under consideration, we cannot simply add v_i to one part of a bipartition (A, B) of G_{i-1}, where G_{i-1} is member of the graph class, and witness that G_i is still a member of the graph class: Adding v_i to A may violate property Π_A *and* adding v_i to B may violate property Π_B. An example for split graph recognition is presented in Fig. 2. Here, we cannot add v_i to A or B to obtain a valid bipartition for G_i, even if G_{i-1} is a split graph with clique A and independent set B. Therefore, we cannot perform a 'compression step' as in iterative compression. Instead, we must attempt to add v_i to each of A and B, and then attempt to 'repair' the resulting partition in each of the two cases, by rearranging vertices, into a solution for G_i (if a solution exists). This idea is formalized in the inductive recognition framework in Section 3.

Monopolar graphs and mutually exclusive graph properties. The first (Π_A, Π_B)-Recognition problem that we consider is the problem of recognizing monopolar graphs, which are a superset of split graphs. A *monopolar graph* is a graph whose vertex set admits a bipartition into a cluster graph and an independent set; an example is shown in Fig. 3. Monopolar graphs have applications in the analysis of protein-interaction networks [4]. The recognition problem of monopolar graphs can be formulated as follows:

> Monopolar Recognition
> **Input:** A graph $G = (V, E)$.
> **Question:** Does G have a *monopolar partition* (A, B), that is, can V be partitioned into sets A and B such that $G[A]$ is a cluster graph and $G[B]$ is an edgeless graph?

In contrast to the recognition problem of split graphs, which admits a linear-time algorithm [16], Monopolar Recognition is NP-hard. This motivates a parameterized complexity analysis of Monopolar Recognition. We consider the parameterized version of Monopolar Recognition, where the parameter k is an upper bound on the number of clusters in $G[A]$, and use inductive recognition to show the following: